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Flexible Backup Supply and the Management of Lead-Time Uncertainty

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1 Appendix

Proof of Proposition 1. 1) With a little algebra, we can get, if $1 > r > 0$ holds, then

$$\begin{aligned} R(\beta|r) &= \frac{1}{2}\frac{(\pi+h)^2-h^2}{\pi+h}\left(\beta T - \frac{(\pi r T-(c_f-c_d))}{(\pi+h)^2-h^2}(\pi+h)\right)^2 + \frac{1}{2}\pi r^2 T^2 \\ &\quad + \frac{1}{2}h(1-r)^2T^2 - \frac{1}{2}\frac{(\pi r T-(c_f-c_d))^2}{(\pi+h)^2-h^2}(\pi+h) \end{aligned}$$

and if $r \geqslant 1$ holds, then

$$\begin{aligned} R(\beta|r) &= \frac{1}{2}\frac{(\pi+h)^2-h^2}{\pi+h}\left(\beta T - \frac{(\pi r T-(c_f-c_d))}{(\pi+h)^2-h^2}(\pi+h)\right)^2 + \pi r T^2 \\ &\quad - \frac{1}{2}\pi T^2 - \frac{1}{2}\frac{(\pi r T-(c_f-c_d))^2}{(\pi+h)^2-h^2}(\pi+h) \end{aligned}$$

Therefore $R(\beta|r)$ is minimized when $\beta T - \frac{(\pi r T-(c_f-c_d))}{(\pi+h)^2-h^2}(\pi+h) = 0$. This leads to our part 1) conclusion in view of the boundary conditions for β.

2) $\beta^* > 0$ hold if and only if $\pi r T - (c_f - c_d) > 0$; and $\beta^* < 1$ hold if and only if $\frac{(\pi r T-(c_f-c_d))}{(\pi+h)^2-h^2}(\pi+h) < T$. This leads to our part 2) conclusion.

3) Part 3) conclusion is true because $(c_f - c_d) > 0$ and $\frac{\pi+h}{\pi+2h} < 1$. ■

Proof of Algorithm 1. We first examine the situations where $\underline{l_f} \leqslant T$ holds. We will analyze the cases defined in (8). For the case $\beta < \frac{\underline{l_f}}{T}, r \leqslant 1$, since it is obvious that the optimal l_f is $\underline{l_f}$, we focus on the decision for β. It can be seen that $R\left(\beta, l_f^*|r\right)$ is linear in β with the first order derivative

$$\frac{\partial R\left(\beta, l_f^* \left| r\right.\right)}{\partial \beta} = T\left(-\pi\left(rT - \underline{l_f}\right) + (c_f - c_d)\right)$$

Thus, when $rT \leqslant \frac{c_f - c_d}{\pi} + \underline{l_f}$, the optimal β is 0; when $rT > \frac{c_f - c_d}{\pi} + \underline{l_f}$, the optimal β is $\frac{\underline{l_f}}{T}$.

For the case $\beta \geqslant \frac{\underline{l_f}}{T}, r \leqslant 1$, it can be seen that $R\left(\beta, l_f \left| r\right.\right)$ is convex in l_f with the first order derivative

$$\frac{\partial R\left(\beta, l_f \left| r\right.\right)}{\partial l_f} = (\pi + h)\, l_f - h\beta T$$

Therefore the decision rule on l_f for given β is: to choose $l_f = \frac{h}{\pi+h}\beta T$ if $\frac{h}{\pi+h}\beta T \geqslant \underline{l_f}$, and to choose $\underline{l_f}$ otherwise. The value of $R\left(\beta, l_f \left| r\right.\right)$ at the optimal l_f, denoted by $R\left(\beta, l_f^* \left| r\right.\right)$, is accordingly given below

$$\begin{aligned} R\left(\beta, l_f^* \left| r\right.\right) &= (c_f - c_d)\,\beta T + \frac{1}{2}\pi\left(rT - \beta T\right)^2 + \frac{1}{2}h\left(T - rT\right)^2 \\ &+ \begin{cases} \frac{1}{2}\pi\left(\underline{l_f}\right)^2 + \frac{1}{2}h\left(\beta T - \underline{l_f}\right)^2 & \text{if } \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T < \underline{l_f}, r \leqslant 1 \\ \frac{1}{2}\frac{\pi h}{\pi+h}\left(\beta T\right)^2 & \text{if } \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T \geqslant \underline{l_f}, r \leqslant 1 \end{cases} \end{aligned}$$

The first order derivative for $R\left(\beta, l_f^* \left| r\right.\right)$ with respect to β can be obtained as follows

$$\frac{dR\left(\beta, l_f^* \left| r\right.\right)}{d\beta} = \begin{cases} \left((c_f - c_d) + (\pi + h)\,\beta T - h\underline{l_f} - \pi rT\right)T & \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T < \underline{l_f}, r \leqslant 1 \\ \left((c_f - c_d) + \frac{\pi h}{\pi+h}\beta T + \pi\beta T - \pi rT\right)T & \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T \geqslant \underline{l_f}, r \leqslant 1 \end{cases}$$

Based on the expression above, it can be seen that with a little algebra, $R\left(\beta, l_f^* \left| r\right.\right)$ is convex in β over $[0, r]$ for given r. Therefore the optimal β can be determined from the first order condition given above. Particularly, we have: a) if $rT < \frac{c_f - c_d}{\pi} + \underline{l_f}$, then the optimal β is 0. This is because $\frac{dR\left(\beta, l_f^* \left| r\right.\right)}{d\beta} > 0$ for $\beta \in [0, r]$; b) if rT is greater than $\frac{c_f - c_d}{\pi} + \underline{l_f}$ and less than $\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is $\frac{\pi}{\pi+h}(rT) + \frac{h}{\pi+h}\underline{l_f} - \frac{c_f - c_d}{\pi+h}$, which is less than rT. This is because $\frac{dR\left(\beta, l_f^* \left| r\right.\right)}{d\beta}$ is negative at $\beta = \left(\frac{c_f - c_d}{\pi} + \underline{l_f}\right)/T$ and is positive at $\beta = \left(\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}\right)/T$; c) if rT is greater than $\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is $\frac{\pi+h}{(\pi+h)^2 - h^2}\left(\pi rT - (c_f - c_d)\right)$, which is less than rT, since $\frac{dR\left(\beta, l_f^* \left| r\right.\right)}{d\beta}$ is negative at $\beta = \left(\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}\right)/T$.

Similar spirit above can be applied to analyze the cases for $r > 1$. For the case $\beta < \frac{\underline{l_f}}{T}, r > 1$, it is obvious that the optimal l_f is $\underline{l_f}$. Regarding the decision for β, we can get: if $rT \leqslant \frac{c_f - c_d}{\pi} + \underline{l_f}$ holds, then the optimal β is 0; if $rT > \frac{c_f - c_d}{\pi} + \underline{l_f}$ holds, then the optimal β is $\frac{\underline{l_f}}{T}$.

For the case $\beta \geqslant \frac{\underline{l_f}}{T}, r > 1$, the optimal l_f is $\underline{l_f}$ if $\frac{h}{\pi+h}\beta T < \underline{l_f}$ holds, and is $\frac{h}{\pi+h}\beta T$ otherwise.

The value of $R(\beta, l_f|r)$ at the optimal l_f, denoted by $R\left(\beta, l_f^*|r\right)$, is

$$R\left(\beta, l_f^*|r\right) = (c_f - c_d)\beta T + \begin{cases} \frac{1}{2}\pi\left(\underline{l_f}\right)^2 + \pi\left(\underline{l_f} - \beta T\right)\left(rT - \underline{l_f}\right) + \frac{1}{2}\pi\left(T - \underline{l_f}\right)^2 + \pi(rT - T)\left(T - \underline{l_f}\right) & \text{if } \beta < \frac{\underline{l_f}}{T}, r > 1 \\ \frac{1}{2}\pi(T - \beta T)^2 + \pi(T - \beta T)(rT - T) + \frac{1}{2}\pi\left(\underline{l_f}\right)^2 + \frac{1}{2}h\left(\beta T - \underline{l_f}\right)^2 & \text{if } \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T < \underline{l_f}, r > 1 \\ \frac{1}{2}\pi(T - \beta T)^2 + \pi(T - \beta T)(rT - T) + \frac{1}{2}\frac{\pi h}{\pi+h}(\beta T)^2 & \text{if } \beta T \geqslant \underline{l_f}, \frac{h}{\pi+h}\beta T \geqslant \underline{l_f}, r > 1 \end{cases}$$

Based on the expression above, we can get the decision of the optimal βT. Particularly, we have: if $rT < \frac{c_f - c_d}{\pi} + \underline{l_f}$, then the optimal βT is 0 ; if rT is between $\frac{c_f - c_d}{\pi} + \underline{l_f}$ and $\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is $\frac{\pi}{\pi+h}(rT) + \frac{h}{\pi+h}\underline{l_f} - \frac{c_f - c_d}{\pi+h}$; if rT is greater than $\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is $\frac{\pi+h}{(\pi+h)^2 - h^2}(\pi rT - (c_f - c_d))$. All of the optimal βT have to be bounded above by T. Particularly, in case $\frac{\pi+h}{(\pi+h)^2 - h^2}(\pi rT - (c_f - c_d)) > T$ and $rT > \frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is T with a cost of $(c_f - c_d)T + \frac{1}{2}\frac{\pi h}{\pi+h}T^2$ for $R\left(\beta^*, l_f^*|r\right)$ if $\frac{h}{\pi+h}T \geqslant \underline{l_f}$; and the optimal βT is T with a cost of $(c_f - c_d)T + \frac{1}{2}\pi\left(\underline{l_f}\right)^2 + \frac{1}{2}h\left(T - \underline{l_f}\right)^2$ for $R\left(\beta^*, l_f^*|r\right)$ if $\frac{h}{\pi+h}T < \underline{l_f}$. In case that $\frac{\pi}{\pi+h}(rT) + \frac{h}{\pi+h}\underline{l_f} - \frac{c_f - c_d}{\pi+h} > T$ and rT is between $\frac{c_f - c_d}{\pi} + \underline{l_f}$ and $\frac{c_f - c_d}{\pi} + \frac{(\pi+h)^2 - h^2}{\pi h}\underline{l_f}$, then the optimal βT is T with a cost of $(c_f - c_d)T + \frac{1}{2}\pi\left(\underline{l_f}\right)^2 + \frac{1}{2}h\left(T - \underline{l_f}\right)^2$ for $R\left(\beta^*, l_f^*|r\right)$.

We now examine the situations where $\underline{l_f} > T$ holds. It is obvious that the optimal l_f is $\underline{l_f}$. Recall that

$$R(\beta, l_f|r) = (c_f - c_d)\beta T + \frac{1}{2}\pi(\beta T)^2 + \pi(\beta T)\left(\underline{l_f} - \beta T\right) + \frac{1}{2}\pi(T - \beta T)^2 + \pi(T - \beta T)(rT - T)$$

It can be seen that with a little algebra, if $rT \leqslant \frac{c_f - c_d}{\pi} + \underline{l_f}$, then the optimal βT is zero with a cost of $\frac{1}{2}\pi T^2 + \pi T(rT - T)$ for $R\left(\beta^*, l_f^*|r\right)$; if $rT > \frac{c_f - c_d}{\pi} + \underline{l_f}$, then the optimal βT is T with a cost of $(c_f - c_d)T + \frac{1}{2}\pi T^2 + \pi T\left(\underline{l_f} - T\right)$ for $R\left(\beta^*, l_f^*|r\right)$.

Putting all the above together yields the proof for Algorithm 1. ■

Proof of Proposition 2. With a little algebra, we can decompose $\overline{V}_{\rm II}\left(Q_1, Q_2|\,\xi_0, l, \widetilde{T}\right)$ as follows

$$\overline{V}_{\rm II}\left(Q_1, Q_2|\,\xi_0, l, \widetilde{T}\right) = \overline{V}_{\rm II}^1\left(Q_1|\,\xi_0, l, \widetilde{T}\right) + \overline{V}_{\rm II}^2\left(Q_2|\,\xi_0, l, \widetilde{T}\right) + (c_f - c_n)\left(T - \widetilde{T}\right) \tag{13}$$

where

$$\overline{V}_{\rm II}^1\left(Q_1|\,\xi_0, l, \widetilde{T}\right) = (c_f - c_d)Q_1 + \frac{1}{2}\frac{\pi h}{\pi + h}Q_1^2 + \frac{1}{2}\pi(\xi_0 - Q_1)^2 \tag{14}$$

$$\overline{V}_{\rm II}^2\left(Q_2|\,\xi_0,l,\widetilde{T}\right) = -\left(c_f-c_d\right)Q_2+\frac{1}{2}\frac{\pi h}{\pi+h}\left(T-\widetilde{T}-Q_2\right)^2 + \begin{cases} \frac{1}{2}h\left(\widetilde{T}+Q_2-\xi_0\right)^2 & \text{if } Q_1<\xi_0\leqslant\widetilde{T}+Q_2 \\ -\frac{1}{2}\pi\left(\widetilde{T}+Q_2-\xi_0\right)^2 & \text{if } Q_1\leqslant\widetilde{T}+Q_2<\xi_0 \end{cases} \tag{15}$$

The first order derivatives of $\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)$ with respect to Q_1 and Q_2 are, respectively,

$$\frac{\partial\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)}{\partial Q_1}=\left(c_f-c_d\right)+\frac{\pi h}{\pi+h}Q_1+\pi\left(Q_1-\xi_0\right) \tag{16}$$

$$\frac{\partial\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)}{\partial Q_2} = -\left(c_f-c_d\right)+\frac{\pi h}{\pi+h}\left(\widetilde{T}+Q_2-T\right) + \begin{cases} h\left(\widetilde{T}+Q_2-\xi_0\right) & \text{if } Q_1<\xi_0\leqslant\widetilde{T}+Q_2 \\ -\pi\left(\widetilde{T}+Q_2-\xi_0\right) & \text{if } Q_1\leqslant\widetilde{T}+Q_2<\xi_0 \end{cases} \tag{17}$$

Based on the expressions above (13), (14), (15), (16) and (17), we see that the following properties hold: 1) $\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)$ is separable in Q_1 and Q_2; and, $\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)$ is convex in Q_1; 2) $\overline{V}_{\rm II}\left(Q_1,Q_2|\,\xi_0,l,\widetilde{T}\right)$ is concave in Q_2 for $\widetilde{T}+Q_2<\xi_0$ and is convex in Q_2 for $Q_1<\xi_0\leqslant\widetilde{T}+Q_2$. Furthermore, by the expressions for $Q_1\left(\xi_0\right)$ and $Q_2\left(\xi_0\right)$ and the expressions above, it can be seen that $Q^{UC}\widehat{=}\left(Q_1\left(\xi_0\right),Q_2\left(\xi_0\right)\right)$ is the unique local minimizer of (10) without constraints.

If $Q_1=Q_2$, then the first-order derivative of $\overline{V}_{\rm II}\left(Q_2,Q_2|\,\xi_0,l,\widetilde{T}\right)$ is

$$\frac{\partial\overline{V}_{\rm II}\left(Q_2,Q_2|\,\xi_0,l,\widetilde{T}\right)}{\partial Q_2} = \frac{\pi h}{\pi+h}Q_2+\frac{\pi h}{\pi+h}\left(\widetilde{T}+Q_2-T\right)+\pi\left(Q_2-\xi_0\right) + \begin{cases} h\left(\widetilde{T}+Q_2-\xi_0\right) & \text{if } Q_2<\xi_0\leqslant\widetilde{T}+Q_2 \\ -\pi\left(\widetilde{T}+Q_2-\xi_0\right) & \text{if } Q_2\leqslant\widetilde{T}+Q_2<\xi_0 \end{cases} \tag{18}$$

The expression above implies that $\overline{V}_{\rm II}\left(Q_2,Q_2|\,\xi_0,l,\widetilde{T}\right)$ is piecewise convex in Q_2. Based on (18), we can get the expression for the minimizer of $\overline{V}_{\rm II}\left(Q_2,Q_2|\,\xi_0,l,\widetilde{T}\right)$. This turns out that $Q^{OA}\widehat{=}\left(Q_2^{OA},Q_2^{OA}\right)$ is the minimizer of $\Gamma_{\rm II}\left(Q_2,Q_2|\,\xi_0,l,\widetilde{T}\right)$.

If $Q_1=\widetilde{T}+Q_2$, then $\overline{V}_{\rm II}\left(\widetilde{T}+Q_2,Q_2\middle|\,\xi_0,l,\widetilde{T}\right)$ has an expression

$$\left(c_f-c_d\right)\widetilde{T}+\left(c_f-c_n\right)\left(T-\widetilde{T}\right)+\frac{1}{2}\frac{\pi h}{\pi+h}\left(\widetilde{T}+Q_2\right)^2+\frac{1}{2}\frac{\pi h}{\pi+h}\left(T-\widetilde{T}-Q_2\right)^2$$

which is convex in Q_2. It can be easily verified that $Q^{BC} \widehat{=} \left(\widetilde{T} + Q_2^{BC}, Q_2^{BC}\right)$ is the minimizer of $\overline{V}_{\rm II}\left(\widetilde{T} + Q_2, Q_2 \middle| \xi_0, l, \widetilde{T}\right)$. Similarly it can be shown that if $Q_2 = 0$, then $\overline{V}_{\rm II}\left(Q_1, 0 | \xi_0, l, \widetilde{T}\right)$ is minimized at $Q^{CO} \widehat{=} \left(Q_1^{CO}, 0\right)$ satisfying

$$Q_1^{CO} = \begin{cases} 0 & \text{if } \pi\xi_0 \leqslant (c_f - c_d) \\ \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} & \text{if } 0 \leqslant \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} \leqslant \widetilde{T} \\ \widetilde{T} & \text{if } \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} \geqslant \widetilde{T} \end{cases}$$

, and that if $Q_2 = T - \widetilde{T}$, then $\overline{V}_{\rm II}\left(Q_1, T - \widetilde{T} \middle| \xi_0, l, \widetilde{T}\right)$ is minimized at $Q^{AB} = \left(Q_1^{AB}, T - \widetilde{T}\right)$ satisfying

$$Q_1^{AB} = \begin{cases} T - \widetilde{T} & \text{if } \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} \leqslant T - \widetilde{T} \\ \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} & \text{if } T - \widetilde{T} \leqslant \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} \leqslant T \\ T & \text{if } \frac{\pi\xi_0 - \left(c_f - c_d\right)}{\frac{\pi h}{\pi + h} + \pi} \geqslant T \end{cases}$$

Now, we are ready to show Proposition 2 is valid.

1). Since $\xi_0 \geqslant T$, $Q_2 + \widetilde{T} \leqslant \xi_0$ holds for any $Q_2 \leqslant T - \widetilde{T}$. Thus $\overline{V}_{\rm II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ is concave in Q_2. For any Q_1, $\overline{V}_{II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ could be minimized only at the boundary points of the feasible set $OABC$. The minimum of $\overline{V}_{II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ could be achieved only at the four sides of the feasible set $OABC$ illustrated in Figure **??**. Since the minimum of $\overline{V}_{II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ on the four sides could only be achieved at one of the four points Q^{OA}, Q^{CO}, Q^{BC} and Q^{AB}, respectively, part 1) follows.

2). Since $\xi_0 < T$, there may exist Q_2 such that $Q_2 + \widetilde{T} > \xi_0$ holds. Thus $\overline{V}_{II}^2\left(Q_2 | \xi_0, l, \widetilde{T}\right)$ is concave-convex in Q_2. For any Q_1, $\overline{V}_{II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ could be minimized only at the boundary points of the feasible set $OABC$ or $Q_2(\xi_0)$. If $Q^{UC} \widehat{=} \left(Q_1(\xi_0), Q_2(\xi_0)\right)$ falls outside the feasible set $OABC$, then any interior point is dominated by some point on the four sides of the feasible region: $\overline{OA}$, $\overline{CO}$, $\overline{BC}$ and $\overline{AB}$; therefore, the minimum of $\overline{V}_{II}\left(Q_1, Q_2 | \xi_0, l, \widetilde{T}\right)$ could only be achieved at one of the four points Q^{OA}, Q^{CO}, Q^{BC} and Q^{AB}. If $Q^{UC} \widehat{=} \left(Q_1(\xi_0), Q_2(\xi_0)\right)$ is an interior point of the feasible set $OABC$, then any interior point is dominated by either Q^{UC} or some point on the four sides. Thus, part 2) follows. ■

Modeling parameters and their values for all the numerical examples

Figure	Modeling parameters values
1.	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3, c_f - c_d = 2$
2.a, 2.b	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3$
3.	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3$ or $5, c_f - c_d = 2$
4.a, 4.b	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3$ or $5, c_f - c_d = 2$
6.a, 6.b	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3$ or $5, c_f - c_d = 2, c_f - c_n = 1.5$
7.a, 7.b	$\pi = 1.8, h = .3, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3, c_n - c_d = 1, \underline{l_f} = 5$
8	$\pi = 1.8, T = 14, \xi \sim Gamma\left(\mu, \theta\right), \mu = 5, \theta = 3, c_f = 5, c_n = 4, c_d = 3$