Appendix A: Proofs

Proof of Proposition 1: If A and B are compatible, network benefits apply regardless of which technology consumers adopt. Consumers will adopt one of the two since $\gamma_i^A, \bar{\gamma}_i^B \geq 0$, $i = 1, 2$. To capture segment i, firm A can charge a maximum price of $p_i^A = \gamma_i^A - \bar{\gamma}_i^B$, leading to profit $(\gamma_i^A - \bar{\gamma}_i^B)x - C^A(\gamma_i^A)$, which is maximized at $\gamma_i^A = \gamma^*$. The corresponding profit is non-negative if and only if $\bar{\gamma}_i^B \leq \gamma^* x - \frac{C^A(\gamma^*)}{x}$. Consumer net benefits are $\bar{\gamma}_i^B + 2\theta x$ regardless of which technology they adopt, therefore, it is not worthwhile for B to develop a product if $\phi + \alpha\bar{\gamma}_1^B x + \beta\bar{\gamma}_2^B x + 4\theta x^2 \leq \bar{c}$. This completes the proof. ■

Proof of Proposition 2: We first derive the equilibrium pricing and product strategies of firm A when B is available first and A and B are incompatible. We can then derive the equilibrium adoption patterns and hence determine when B would develop a product. We then compare the profits of A to Proposition 1 to obtain the second result of the Proposition. Let $\Delta_i \equiv \gamma_i^A - \bar{\gamma}_i^B$. Given the feature benefits of firm A, (γ_1^A, γ_2^A), the necessary and sufficient conditions for AA, BB, AB, or BA to be an equilibrium are

$$AA: \quad p_1^A \leq \Delta_1 + \theta x \quad \text{and} \quad p_2^A \leq \Delta_2 + \theta x, \tag{2}$$

$$BB: \quad p_1^A \geq \Delta_1 - \theta x \quad \text{and} \quad p_2^A \geq \Delta_2 - \theta x, \tag{3}$$

$$AB: \quad p_1^A \leq \Delta_1 - \theta x \quad \text{and} \quad p_2^A \geq \Delta_2 + \theta x, \tag{4}$$

$$BA: \quad p_1^A \geq \Delta_1 + \theta x \quad \text{and} \quad p_2^A \leq \Delta_2 - \theta x. \tag{5}$$

AA and BB are both equilibria if

$$\Delta_1 - \theta x \leq p_1^A \leq \Delta_1 + \theta x \quad \text{and} \quad \Delta_2 - \theta x \leq p_2^A \leq \Delta_2 + \theta x. \tag{6}$$

If (6) holds, we assume consumers choose the equilibrium with the higher total consumer net benefits, i.e., they choose AA if

$$\Delta_1 + \Delta_2 \geq p_1^A + p_2^A, \tag{7}$$

otherwise, they choose BB. Given consumers' period-2b equilibrium strategies (equations (2)-(7)), we now derive the period-2a pricing and feature benefit strategies of firm A. We define the following sets in $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$ space:

$$
\begin{aligned}
D_{AB} &\equiv \left\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \middle| \bar{\gamma}_1^B \le \gamma^* - \theta x - \tfrac{C^A(\gamma^*)}{x} \text{ and } \bar{\gamma}_2^B > \gamma^* + \theta x - \tfrac{C^A(\gamma^*)}{x} \right\}, \\
D_{BA} &\equiv \left\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \middle| \bar{\gamma}_1^B > \gamma^* + \theta x - \tfrac{C^A(\gamma^*)}{x} \text{ and } \bar{\gamma}_2^B \le \gamma^* - \theta x - \tfrac{C^A(\gamma^*)}{x} \right\}, \\
D_{BB} &\equiv \Big\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \Big| \bar{\gamma}_1^B > \gamma^* - \theta x - \tfrac{C^A(\gamma^*)}{x} \text{ and } \bar{\gamma}_2^B > \gamma^* - \theta x - \tfrac{C^A(\gamma^*)}{x} \\
&\qquad \text{and } (\gamma^* - \bar{\gamma}_1^B)x + (\gamma^* - \bar{\gamma}_2^B)x - 2C^A(\gamma^*) < 0 \Big\}, \text{ and} \\
D_{AA} &\equiv \text{the rest of the first quadrant in } (\bar{\gamma}_1^B, \bar{\gamma}_2^B) \text{ space.}
\end{aligned}
$$

The sets D_{AB}, D_{BA} D_{BB}, and D_{AA}, correspond to the regions in Figure 2 where AB, BA, BB, and AA are the equilibrium adoption patterns, respectively.

Lemma 3 *When B is available first and A and B are incompatible, the subgame perfect equilibrium is for consumers to follow strategies (2)-(7) and for firm A to set the following feature benefits and prices:*

- *Split market equilibrium: Let $TT' = AB$ or BA and segment $i \neq j$ be the segment firm A wins. If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{TT'}$, firm A sets its feature benefits at $\tilde{\gamma}_i^A = \gamma^*$, $\tilde{\gamma}_j^A = 0$, and prices $\tilde{p}_i^A = \gamma^* - \bar{\gamma}_i^B - \theta x$, $\tilde{p}_j^A = 0$, resulting in adoption pattern TT', profit $\tilde{p}_i^A x - C^A(\gamma^*)$, and consumer net benefits $\bar{\gamma}_i^B + 2\theta x$ for segment i, and $\bar{\gamma}_j^B + \theta x$ for segment j.*

- *Technology B wins: If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BB}$, for each segment $i = 1, 2$, firm A sets its feature benefits at $\tilde{\gamma}_i^A = 0$, and prices $\tilde{p}_i^A = 0$, resulting in adoption pattern BB, zero profit, and consumer net benefits $w_i^{BB} = \bar{\gamma}_i^B + 2\theta x$.*

- *Firm A wins: If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$, for each segment $i = 1, 2$, firm A sets its feature benefits at $\tilde{\gamma}_i^A = \gamma^*$ and prices $\tilde{p}_i^A = \gamma^* - \bar{\gamma}_i^B$, resulting in adoption pattern AA, profit $\Pi_{AA} = (\tilde{p}_1^A + \tilde{p}_2^A)x - 2C^A(\gamma^*)$, and consumer net benefits $w_i^{AA} = \bar{\gamma}_i^B + 2\theta x$.*

Proof : We compare the optimal product and pricing strategies of firm A when it wins both segments and when it wins only one segment, to determine which case applies. Consider the pricing decision of firm A given the feature benefits, (γ_1^A, γ_2^A), and strategies of consumers in period $2b$, (2)-(7). The maximum profit firm A can make from winning both segments is $(\Delta_1 + \Delta_2)x - C^A(\gamma_1^A) - C^A(\gamma_2^A)$ (from (2) and (6)), which is maximized at (γ^*, γ^*). Therefore, firm A's maximum profit from winning both segments is $\Pi_{AA} = (\gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B)x - 2C^A(\gamma^*)$.

Given the feature benefits (γ_1^A, γ_2^A), firm A's maximum profit from winning only segment 1 is $\Delta_1 x - \theta x^2 - C^A(\gamma_1^A) - C^A(\gamma_2^A)$ (from (4)), which is maximized at $(\gamma^*, 0)$. Therefore, firm A's maximum profit from winning only segment 1 is $\Pi_{AB} = (\gamma^* - \bar{\gamma}_1^B)x - \theta x^2 - C^A(\gamma^*)$. A symmetric analysis shows that the profit-maximizing feature benefits when firm A wins only segment 2 is $(0, \gamma^*)$, resulting in maximum profit of $\Pi_{BA} = (\gamma^* - \bar{\gamma}_2^B)x - \theta x^2 - C^A(\gamma^*)$.

It is profit maximizing for A to price to win both segments if and only if $\Pi_{AA} \geq \max\{\Pi_{AB}, \Pi_{BA}, \Pi_{BB}\}$, which defines D_{AA}. There is a range of prices, $\{(\tilde{p}_1^A, \tilde{p}_2^A) | \tilde{p}_1^A \in [\gamma^* - \bar{\gamma}_1^B - \theta x, \gamma^* - \bar{\gamma}_1^B + \theta x], \tilde{p}_2^A = \gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B - \tilde{p}_1^A\}$, that satisfy (7) and give the maximum Π_{AA}, however, only $\tilde{p}_1^A = \gamma^* - \bar{\gamma}_1^B$ and $\tilde{p}_2^A = \gamma^* - \bar{\gamma}_2^B$ result in AA being the pareto-optimal period-$2b$ equilibrium. It is profit maximizing for firm A to price to win only segment 1 if and only if $\Pi_{AB} \geq \max\{\Pi_{AA}, \Pi_{BA}, \Pi_{BB}\}$. It is profit maximizing for firm A to price to win only segment 2 if and only if $\Pi_{BA} \geq \max\{\Pi_{AA}, \Pi_{AB}, \Pi_{BB}\}$. Now, $\Pi_{AB} \geq \Pi_{AA}$ and $\Pi_{BA} \geq \Pi_{BB}$ together imply $\Pi_{AB} \geq \Pi_{BA}$. Therefore, $\Pi_{AB} \geq \max\{\Pi_{AA}, \Pi_{BB}\}$ defines D_{AB} and $\Pi_{BA} \geq \max\{\Pi_{AA}, \Pi_{BB}\}$ defines D_{BA}, the two split market equilibria. Under all other conditions, technology B wins both segments, $D_{BB} = (\bar{\gamma}_1^B, \bar{\gamma}_2^B) \backslash D_{AA} \cup D_{AB} \cup D_{BA}$. This completes the proof. ■

If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA} \cup D_{BB}$, consumer net benefits are $\bar{\gamma}_i^B + 2\theta x$, $i = 1, 2$, therefore, B enters the market if and only if $\phi + \alpha\bar{\gamma}_1^B x + \beta\bar{\gamma}_2^B x + 4\theta x^2 \geq \bar{c}$. If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AB} \cup D_{BA}$, consumer net benefits are $\bar{\gamma}_i^B + \theta x$ and $\bar{\gamma}_j^B + 2\theta x$, $i = 1, 2$, $i \neq j$, therefore, B enters the market if and only if $\phi + \alpha\bar{\gamma}_1^B x + \beta\bar{\gamma}_2^B x + (\alpha + 2\beta)\theta x^2 \geq \bar{c}$. Comparing B's entry criteria with Proposition 1 shows that B is less likely to develop a product when A and B are incompatible than when they are compatible. The second part of the Proposition is a direct result of comparing Lemma 3 and Proposition 1. ■

Proof of Proposition 3: The four adoption patterns yield the following expressions for the optimal overall surplus: $Y^{AA} = 2\gamma^* x + 4\theta x^2 - 2C^A(\gamma^*)$ ($C^B = 0$ to maximize total network value), $Y^{AB} = (\gamma^* + \bar{\gamma}_2^B)x + 2\theta x^2 - C^A(\gamma^*) + \phi - \bar{c}$, $Y^{BA} = (\bar{\gamma}_1^B + \gamma^*)x + 2\theta x^2 - C^A(\gamma^*) + \phi - \bar{c}$, and $Y^{BB} = (\bar{\gamma}_1^B + \bar{\gamma}_2^B)x + 4\theta x^2 + \phi - \bar{c}$. TT' is socially optimal if $Y^{TT'} = \max\{Y^{AA}, Y^{AB}, Y^{BA}, Y^{BB}\}$. It is straightforward to show that AA is socially optimal for a superset of D_{AA}, and BB is socially optimal for a superset of D_{BB}. Hence, the region of socially optimal compatibility is a superset of $D_{AA} \cup D_{BB}$, which completes the proof. ■

Proof of Lemma 1: We maximize π_i by solving the following two maximization problems for all values of $\bar{\gamma}_i^B$:

$$\max_{\gamma_i^A} \quad (\gamma_i^A - \bar{\gamma}_i^B)x + \theta x^2 - C^A(\gamma_i^A)$$
$$\text{s.t. } \gamma_i^A > \bar{\gamma}_i^B + \theta x, \tag{8}$$
$$\max_{\gamma_i^A} \quad 2(\gamma_i^A - \bar{\gamma}_i^B)x - C^A(\gamma_i^A)$$
$$\text{s.t. } 0 \le \gamma_i^A \le \bar{\gamma}_i^B + \theta x. \tag{9}$$

We can easily show that γ^* solves the unconstrained version of (8), and γ^{**} the unconstrained version of (9). Because $\frac{\partial (C^A)^2}{\partial^2 \gamma_i^A} > 0$, we know that $\gamma^* < \gamma^{**}$. If $\bar{\gamma}_i^B + \theta x \le \gamma^* < \gamma^{**}$, then the maximizing values of (8) and (9) are γ^* and $\bar{\gamma}_i^B + \theta x$ (by concavity of (9)). Comparing the respective profits of (8) and (9), we obtain $(\gamma^* - \bar{\gamma}_i^B)x + \theta x^2 - C^A(\gamma^*) \ge 2\theta x^2 - C^A(\bar{\gamma}_i^B + \theta x)$ (because γ^* maximizes $\gamma_i^A x - C^A(\gamma_i^A)$). Therefore, the profits in (8) are greater than or equal to those in (9), which proves the result for $\bar{\gamma}_i^B \le \gamma^* - \theta x$.

If $\gamma^* < \bar{\gamma}_i^B + \theta x \le \gamma^{**}$, then the maximizing values of (8) and (9) are both $\bar{\gamma}_i^B + \theta x$ (by concavity of (8) and 9)). The profits in (8) and (9) are equal: $2\theta x^2 - C^A(\bar{\gamma}_i^B + \theta x) = 2\theta x^2 - C^A(\bar{\gamma}_i^B + \theta x)$, which proves the result for $\gamma^* < \bar{\gamma}_i^B + \theta x \le \gamma^{**}$.

If $\bar{\gamma}_i^B + \theta x > \gamma^{**}$, then the maximizing values of (8) and (9) are $\bar{\gamma}_i^B + \theta x$ (by concavity of (8)) and γ^{**}. Comparing the respective profits in (8) and (9), $2\theta x^2 - C^A(\bar{\gamma}_i^B + \theta x) \le 2(\gamma^{**} - \bar{\gamma}_i^B)x - C^A(\gamma^{**})$ (because γ^{**} maximizes $2\gamma_i^A x - C^A(\gamma_i^A)$). Therefore the profits in (8) are less than or equal to those in (9), which gives the results for $\bar{\gamma}_i^B > \gamma^{**} - \theta x$, and completes the proof. ■

Lemma 4 *Firm A has a first-mover advantage by inducing adoption sequence (12) under the following conditions:*

(i) If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$ *and* $\bar{\gamma}_1^B \le \gamma_{AA}^{(12)}$, *firm A maximizes its profit by setting* $\tilde{\gamma}_1^A = \hat{\gamma}_1^A$, $\tilde{\gamma}_2^A = \gamma^*$, *resulting in profit* $\Pi_{AA}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) + \hat{\pi}_1$.

(ii) If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{BB} \cap E_{BB}^{(12)}$, *firm A maximizes its profit by setting* $\tilde{\gamma}_1^A = \hat{\gamma}_1^A$, $\tilde{\gamma}_2^A = \gamma^*$, *resulting in profit* $\Pi_{BB}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) + \hat{\pi}_1$.

(iii) If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AB} \cap E_{AB}^{(12)}$, *firm A maximizes its profit by setting* $\tilde{\gamma}_1^A = \gamma^*$, $\tilde{\gamma}_2^A = \gamma^*$, *resulting in profit* $\Pi_{AB}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) + \hat{\pi}_1$.

(iv) If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{BA}$ *and* $\bar{\gamma}_1^B \le \gamma_{BA}^{(12)}$, *firm A maximizes its profit by setting* $\tilde{\gamma}_1^A = \hat{\gamma}_1^A$, $\tilde{\gamma}_2^A = \gamma^*$, *resulting in profit* $\Pi_{BA}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x + \theta x^2 \cdot 1_{\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BA}\}} - C^A(\gamma^*) + \hat{\pi}_1$,

where $\gamma_{AA}^{(12)} \equiv \min\{\bar{\gamma}_1^B | \hat{\pi}_1 - (\gamma^* - \bar{\gamma}_1^B)x + C^A(\gamma^*) \ge 0\}$, $E_{BB}^{(12)} \equiv \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_1^B \le \hat{\gamma}_1^A + \theta x$ *and* $\hat{\pi}_1 + (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) \ge 0\}$, $E_{AB}^{(12)} \equiv \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) + \hat{\pi}_1 \ge (\gamma^* - \bar{\gamma}_1^B)x - C^A(\gamma^*)\}$, $\gamma_{BA}^{(12)} \equiv$

$\min\{\hat{\gamma}_1^A + \theta x, \min\{\bar{\gamma}_1^B | \hat{\pi}_1 + \theta x^2 \cdot 1_{\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BA}\}} \geq 0\}\}$, $E_{TT'} \equiv \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \Gamma_1$ *and* $\Gamma_2\}$, *and* Γ_i *represents the condition* $\bar{\gamma}_i^B \leq \gamma^* - \frac{C^A(\gamma^*)}{x}$ *if segment* i *adopts technology* A, *and* $\bar{\gamma}_i^B > \gamma^* - \frac{C^A(\gamma^*)}{x}$ *if segment* i *adopts technology* B. *The resulting adoption pattern when firm* A *has a first-mover advantage is* AA. *Firm* A*'s first-mover advantage under sequence* (21) *is analogously defined by switching subscripts* 1 *and* 2 *in (i) -(iv) and subscripts* A *and* B *in (iii)-(iv).*

Proof of Lemma 4: We compare firm A's profit when A and B are incompatible, A designs optimally and prices to induce sequence (12), to its maximum profit when B is available first (when A and B are compatible). Let $\Pi_{TT'}$ be firm A's profit when A and B are compatible and $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{TT'}$. Consider the period-2$c$ subgame when only segment 1 has adopted A in period 1, firm A has made product and pricing decisions (γ_1^A, γ_2^A) and $(p_1^A(2), p_2^A(2))$, and technologies A and B are incompatible.

¿From the payoff matrix in Figure 3(a), we can derive the necessary and sufficient conditions for each adoption pattern to be an equilibrium. AA and BB are both equilibria if $-\theta x \leq \Delta_1 \leq \theta x$ and $\Delta_2 - \theta x \leq p_2^A(2) \leq \Delta_2 + \theta x$, in which case consumers choose the equilibrium with the higher total consumer net benefits, i.e., they choose AA if $p_2^A(2) \leq \Delta_1 + \Delta_2$, and BB otherwise. If firm A prices to win both segments, then $p_2^A(2) \leq \Delta_2 + \theta x$ (necessary condition for AA) and $p_2^A(2) \leq \Delta_1 + \Delta_2$ imply

$$p_2^A(2) = \begin{cases} \Delta_2 + \theta x, & \Delta_1 \geq \theta x \\ \Delta_1 + \Delta_2, & \Delta_1 < \theta x \end{cases}. \tag{10}$$

From (13a), to induce segment-1 consumers to adopt in period 1, $w_1^{(12)} \geq w_1^{(22)}$.

Suppose $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$. From Lemma 3, $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$ and from Proposition 1, $\Pi_{AA} = (\gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B)x - 2C^A(\gamma^*)$. If firm A sets prices in period 2 so the equilibrium is BB, it will make non-positive profit. If firm A sets prices in period 2 so the equilibrium is AB, $w_1^{(12)} \geq w_1^{(22)}$ implies $p_1^A(1) = \Delta_1 - \theta x$, resulting in profit $\Delta_1 x - \theta x^2 - C^A(\gamma_1^A) - C^A(\gamma_2^A)$ (no profit from segment 2). Profit is maximized at $(\gamma^*, 0)$ resulting in $\Pi_{AB}^{(12)} = (\gamma^* - \bar{\gamma}_1^B)x - \theta x^2 - C^A(\gamma^*) < \Pi_{AB} \leq \Pi_{AA}$ (from Proposition 1, $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$ implies $\Pi_{AA} \geq \Pi_{AB}$). Firm A does not have a first-mover advantage by pricing to induce adoption pattern AB in the period-2c subgame.

If firm A sets prices in period 2 so the equilibrium is BA, $w_1^{(12)} \geq w_1^{(22)}$ implies $p_1^A(1) = -\theta x$, and $p_2^A(2) \leq \Delta_2 - \theta x$ (necessary condition for BA) implies $p_2^A(2) = \Delta_2 - \theta x$, which results in profit $\Delta_2 x - 2\theta x^2 - C^A(\gamma_1^A) - C^A(\gamma_2^A)$. Profit is maximized at $(0, \gamma^*)$ resulting in $\Pi_{BA}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x - 2\theta x^2 - C^A(\gamma^*) < \Pi_{BA} \leq \Pi_{AA}$ (from Proposition 1, $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$ implies $\Pi_{AA} \geq \Pi_{BA}$). Firm A does not have a first-mover advantage by pricing to induce adoption pattern BA in the period-2c subgame.

If firm A sets prices in period 2 so the equilibrium is AA, $w_1^{(12)} \geq w_1^{(22)}$ implies $p_1^A(1) = \Delta_1$, and

(10) defines $p_2^A(2)$, resulting in profit $\Delta_2 x - C^A(\gamma_2^A) + \pi_1$. Profit is maximized at $(\hat{\gamma}_1^A, \gamma^*)$, resulting in $\Pi_{AA}^{(12)} = (\gamma^* - \bar{\gamma}_2^B)x - C^A(\gamma^*) + \hat{\pi}_1 \geq \Pi_{AA}$ if and only if $\hat{\pi}_1 - (\gamma^* - \bar{\gamma}_1^B)x + C^A(\gamma^*) \geq 0$, which is a function of $\bar{\gamma}_1^B$ only. Since $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$, $\bar{\gamma}_1^B \leq \gamma^* - \frac{C^A(\gamma^*)}{x} \leq \hat{\gamma}_1^A - \frac{C^A(\gamma^*)}{x} \leq \hat{\gamma}_1^A + \theta x$, satisfying $\Delta_1 \geq -\theta x$, a necessary condition for the equilibrium to be AA. Therefore, if $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{AA}$, firm A has a first-mover advantage if and only if it prices to induce adoption pattern AA and $\bar{\gamma}_1^B \leq \gamma_{AA}^{(12)} \equiv \min\{\bar{\gamma}_1^B | \hat{\pi}_1 - (\gamma^* - \bar{\gamma}_1^B)x + C^A(\gamma^*) \geq 0\}$. The optimal feature benefits are $(\hat{\gamma}_1^A, \gamma^*)$.

We can similarly find the conditions for first-mover advantage when $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in E_{BB}$, E_{AB}, $E_{BA} \cap D_{BA}$, and $E_{BA} \backslash D_{BA}$. The analysis for first-mover advantage under sequence `(21)` is analogous. ■

Proof of Lemma 2: We compare firm A's maximum profit under sequence `(11)` to its maximum profit under sequences `(12)`, `(21)`, and `(22)`. If its profit under sequence `(11)` is less than or equal to its profit under sequence `(22)`, then from Lemma 4, sequence `(12)` or `(21)` dominates `(11)`. The necessary and sufficient conditions sequences `(11)`, `(22)`, `(12)`, and `(21)` are:

$$w_1^{(11)} \geq w_1^{(21)} \text{ and } w_2^{(11)} \geq w_2^{(12)}, \tag{11}$$

$$w_1^{(22)} \geq w_2^{(12)} \text{ and } w_2^{(22)} \geq w_2^{(21)}, \tag{12}$$

$$w_1^{(12)} \geq w_1^{(22)} \text{ and } w_2^{(12)} \geq w_2^{(11)}, \tag{13}$$

and $$w_1^{(21)} \geq w_1^{(11)} \text{ and } w_2^{(21)} \geq w_2^{(22)}. \tag{14}$$

If (11) and (12) hold, then consumers choose sequence `(11)` if

$$w_1^{(11)} + w_2^{(11)} \geq w_1^{(22)} + w_2^{(22)}, \tag{15}$$

otherwise, they choose sequence `(22)`. Consumers will not pay a positive price to adopt A in period 1 if they expect to switch to B in period 2. Firm A will not subsidize consumers to adopt A in period 1 if they are going to switch to B in period 2, because firm A cannot leverage the network effects to charge a higher price. Therefore, we need only consider the AA equilibrium in the period-2 subgame, i.e., $w_1^{(11)} = \gamma_1^A + 2\theta x$ and $w_2^{(11)} = \gamma_1^A + 2\theta x$ (Figure 3(b)). Since technologies A and B are incompatible, $w_1^{(22)}$ and $w_2^{(22)}$ are as given in Lemma 3. Let $\Pi_{TT'}$ and $\Pi_{TT'}^{(ij)}$ be firm A's profits when B is available first and under sequence `(ij)`, respectively.

Consider first the case when (12) holds, so to induce sequence `(11)`, firm A must price so that (15) is binding. Suppose $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$. From Lemma 3, $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$ and $w_2^{(22)} = \bar{\gamma}_2^B + 2\theta x$. Equation (15) gives us $p_1^A(1) + p_2^A(1) \leq \Delta_1 + \Delta_2$. Firm A's profit is maximized at (γ^*, γ^*), resulting in profit $(\gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B)x - 2C^A(\gamma^*) = \Pi_{AA}$. Suppose $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BB}$. From Lemma 3, $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$ and $w_2^{(22)} = \bar{\gamma}_2^B + 2\theta x$, resulting in profit $(\gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B)x - 2C^A(\gamma^*) = \Pi_{AA} \leq \Pi_{BB}$

(from Lemma 3, $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BB}$ implies $\Pi_{BB} \geq \Pi_{AA}$). Suppose $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AB}$. From Lemma 3, $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$ and $w_2^{(22)} = \bar{\gamma}_2^B + \theta x$. Equation (15) gives us $p_1^A(1) + p_2^A(1) \leq \Delta_1 + \Delta_2 + \theta x$. Firm A's profit is maximized at (γ^*, γ^*), resulting in profit $(\gamma^* - \bar{\gamma}_1^B + \gamma^* - \bar{\gamma}_2^B)x + \theta x^2 - 2C^A(\gamma^*) = \Pi_{AB}^{(12)}$. Suppose $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BA}$. From Lemma 3, $w_1^{(22)} = \bar{\gamma}_1^B + \theta x$ and $w_2^{(22)} = \bar{\gamma}_2^B + 2\theta x$. Equation (15) gives us $p_1^A(1) + p_2^A(1) \leq \Delta_1 + \Delta_2 + \theta x$. Firm A's profit is maximized at (γ^*, γ^*), resulting in profit $(\gamma^* - \bar{\gamma}_1^B + \gamma_2^* - \bar{\gamma}_2^B)x + \theta x^2 - 2C^A(\gamma^*) = \Pi_{BA}^{(21)}$.

Suppose (12) does not hold. If $w_1^{(12)} \geq w_1^{(22)}$, then pricing so that $w_1^{(11)} \geq w_1^{(21)}$ and $w_2^{(11)} \geq w_2^{(12)}$ is sufficient to induce adoption sequence (11). However, $w_1^{(12)} \geq w_1^{(22)}$ and $w_2^{(11)} \geq w_2^{(12)}$ are exactly the same conditions for sequence (12). Therefore, the profit under sequence (11) is equivalent to the profit under sequence (12). The analysis for the case where $w_2^{(21)} \geq w_2^{(22)}$ is analogous. ■

Proof of Proposition 4: The results of this proposition follow directly from Lemmas 4 and 2. We need only show that $F = \{E_{AA} \cap \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_1^B \leq \gamma_{AA}^{(12)} \text{ or } \bar{\gamma}_2^B \leq \gamma_{AA}^{(21)}\}\} \cup \{E_{BB} \cap (E_{BB}^{(12)} \cup E_{BB}^{(21)})\} \cup \{E_{AB} \cap (E_{AB}^{(12)} \cup \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_2^B \leq \gamma_{AB}^{(21)}\})\} \cup \{E_{BA} \cap (\{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_1^B \leq \gamma_{BA}^{(12)}\} \cup E_{BA}^{(21)})\} \neq \emptyset$. Note that from Lemma 1, $\hat{\gamma}_i^A \geq \gamma^*$. Consider the set $E_{AA} \cap \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_1^B \leq \gamma_{AA}^{(12)} \text{ or } \bar{\gamma}_2^B \leq \gamma_{AA}^{(21)}\}$. For $\bar{\gamma}_1^B \leq \gamma^* - \theta x$, $\hat{\pi}_1 - (\gamma^* - \bar{\gamma}_1^B)x + C^A(\gamma^*) \geq 0$ and $\gamma_{AA}^{(12)} = \bar{\gamma}_1^B$. Clearly $E_{AA} \cap \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_1^B \leq \gamma^* - \theta x\} \neq \emptyset$, thereby proving the proposition. We can analogously show that $E_{AA} \cap \{(\bar{\gamma}_1^B, \bar{\gamma}_2^B) | \bar{\gamma}_2^B \leq \gamma^* - \theta x\} \neq \emptyset$. ■

Proof of Proposition 5: This result follows directly from Lemmas 2 and 4. When A has a first-mover advantage, A wins the entire market. This means that when B enters the market in period 2, it is merely acting as a credible threat to force A to lower its prices. If $\phi + w_1^{(12)}x + w_2^{(12)}x < \bar{c}$, then B will not develop a product at all, in which case A will have a second period monopoly. ■

Proof of Proposition 6: We first prove that γ_i^A (weakly) increases in $\bar{\gamma}_i^B$. From Lemma 1, if $\bar{\gamma}_i^B \leq \gamma^* - \theta x$, $\hat{\gamma}_i^A = \gamma^*$ which is constant in $\bar{\gamma}_i^B$. As $\bar{\gamma}_i^B$ increases to the range $\gamma^* - \theta x \leq \bar{\gamma}_i^B \leq \gamma^{**} - \theta x$, $\hat{\gamma}_i^A = \bar{\gamma}_i^B + \theta x \geq \gamma^*$ and increases in $\bar{\gamma}_i^B$. If $\bar{\gamma}_i^B \geq \gamma^{**} - \theta x$, $\hat{\gamma}_i^A = \gamma^{**}$ which is constant in $\bar{\gamma}_i^B$. When firm A has a first-mover advantage, $\tilde{\gamma}_i^A = \gamma^*$, which is constant in $\bar{\gamma}_i^B$, or $\tilde{\gamma}_i^A = \hat{\gamma}_i^A$, which is (weakly) increasing in $\bar{\gamma}_i^B$. This completes the proof for the first part of the proposition.

We next prove that γ_i^A (weakly) increases in θ. Suppose the network effects increase by δ. Then the profit-maximizing γ_i^A for profit function $\pi_i(\gamma_i^A)$ is

$$\check{\gamma}_i^A = \begin{cases} \gamma^*, & \text{if } \bar{\gamma}_i^B \leq \gamma^* - (\theta+\delta)x \\ \bar{\gamma}_i^B + (\theta+\delta)x, & \text{if } \gamma^* - (\theta+\delta)x < \bar{\gamma}_i^B \leq \gamma^{**} - (\theta+\delta)x \\ \gamma^{**}, & \text{if } \bar{\gamma}_i^B > \gamma^{**} - (\theta+\delta)x \end{cases}.$$

Comparing with $\hat{\gamma}_i^A$ from Lemma 1, if $\bar{\gamma}_i^B \leq \gamma^* - (\theta+\delta)x$, $\check{\gamma}_i^A = \hat{\gamma}_i^A = \gamma^*$. If $\gamma^* - (\theta+\delta)x < \bar{\gamma}_i^B \leq \gamma^* - \theta x$,

$\tilde{\gamma}_i^A = \bar{\gamma}_i^B + (\theta+\delta)x > \gamma^* = \hat{\gamma}_i^A$. If $\gamma^* - \theta x < \bar{\gamma}_i^B \leq \gamma^{**} - (\theta+\delta)x$, $\tilde{\gamma}_i^A = \bar{\gamma}_i^B + (\theta+\delta)x > \bar{\gamma}_i^B + \theta x = \hat{\gamma}_i^A$. If $\gamma^{**} - (\theta+\delta)x < \bar{\gamma}_i^B \leq \gamma^{**} - \theta x$, $\tilde{\gamma}_i^A = \gamma^{**} > \bar{\gamma}_i^B + \theta x = \hat{\gamma}_i^A$. If $\bar{\gamma}_i^B > \gamma^{**} - (\theta+\delta)x$, $\tilde{\gamma}_i^A = \hat{\gamma}_i^A = \gamma^{**}$. When firm A has a first-mover advantage, $\tilde{\gamma}_i^A = \gamma^*$, which is constant in θ, or $\tilde{\gamma}_i^A = \hat{\gamma}_i^A$, which is (weakly) increasing in θ. This completes the proof. ■

Proof of Proposition 7: From Lemma 4, when firm A has a first-mover advantage, $\tilde{\gamma}_i^A = \gamma^*$ or $\tilde{\gamma}_i^A = \hat{\gamma}_i^A \geq \gamma^*$ (Lemma 1). When B is available first, $\tilde{\gamma}_i^A = 0$ or $\tilde{\gamma}_i^A = \gamma^*$. Now, $\min\{\gamma^*, \hat{\gamma}_i^A\} \geq \max\{0, \gamma^*\}$, which completes the proof. ■

Proof of Proposition 8: For each $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$, we compare consumer net benefits under sequences `(11)`, `(12)`, `(21)`, and `(22)`, given that firm A acts optimally in period 2. If segment 1 adopts B in period 1, firm A will enter segment 2 in period 2 if and only if it can make non-negative profit, i.e., $\bar{\gamma}_2^B \leq \gamma^* - \theta x - \frac{C^A(\gamma^*)}{x} \equiv \bar{\gamma}$, resulting in adoption pattern BA and consumer net benefits $w_1^{(12)} = \bar{\gamma}_1^B + \theta x$, $w_2^{(12)} = \bar{\gamma}_2^B + 2\theta x$. Similarly, if segment 2 adopts B in period 1, firm A will enter segment 1 in period 2 if and only if $\bar{\gamma}_1^B \leq \bar{\gamma}$, resulting in adoption pattern AB and consumer net benefits $w_1^{(21)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(12)} = \bar{\gamma}_2^B + \theta x$. If both segments adopt B in period 1, the resulting adoption pattern is BB and consumer net benefits are $w_1^{(11)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(11)} = \bar{\gamma}_2^B + 2\theta x$. If neither segment adopts B in period 1, then the competition described in Lemma 3 (when B is available first) is played in period 2.

If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$ and $\bar{\gamma}_1^B \leq \bar{\gamma}$ and $\bar{\gamma}_2^B > \bar{\gamma}$, sequences `(11)` and `(12)` lead to adoption pattern BB and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + 2\theta x$, `(tt')` = `(11)`, `(12)`. Sequence `(21)` leads to adoption pattern AB and consumer net benefits $w_1^{(21)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(21)} = \bar{\gamma}_2^B + \theta x$. Sequence `(22)` leads to adoption pattern AA and consumer net benefits $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(22)} = \bar{\gamma}_2^B + 2\theta x$. Therefore sequences `(11)`, `(12)`, and `(22)` are equilibrium adoption sequences resulting in adoption patterns BB, BB, and AA, respectively. We can similarly show that if $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$ and $\bar{\gamma}_1^B > \bar{\gamma}$ and $\bar{\gamma}_2^B \leq \bar{\gamma}$, sequences `(11)`, `(21)`, and `(22)` are equilibrium adoption sequences resulting in adoption patterns BB, BB, and AA, respectively.

If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$ and $\bar{\gamma}_1^B \leq \bar{\gamma}$ and $\bar{\gamma}_2^B \leq \bar{\gamma}$, sequences `(11)` and `(22)` lead to adoption patterns BB and AA, respectively, and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + 2\theta x$, `(tt')` = `(11)`, `(22)`. Sequence `(21)` leads to adoption pattern AB and consumer net benefits $w_1^{(21)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(21)} = \bar{\gamma}_2^B + \theta x$. Sequence `(12)` leads to adoption pattern BA and consumer net benefits $w_1^{(12)} = \bar{\gamma}_1^B + \theta x$, $w_2^{(12)} = \bar{\gamma}_2^B + 2\theta x$. Therefore, sequences `(11)` and `(22)` are equilibrium adoption sequences resulting in adoption patterns BB and AA, respectively.

If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AA}$ and $\bar{\gamma}_1^B > \bar{\gamma}$ and $\bar{\gamma}_2^B > \bar{\gamma}$, sequences `(11)`, `(12)`, and `(21)` lead to adoption pattern BB and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + 2\theta x$, `(tt')` = `(11)`, `(12)`, `(21)`.

Sequence (22) leads to adoption pattern AA and consumer net benefits $w_1^{(22)} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(22)} = \bar{\gamma}_2^B + 2\theta x$. Therefore, all four adoption sequences are equilibria leading to adoption patterns BB and AA. If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BB}$, all four adoption sequences are equilibria and lead to adoption pattern BB and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + 2\theta x$, (tt') = (11), (12), (21), (22).

If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AB}$, sequences (11) and (12) lead to adoption pattern BB and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + 2\theta x$, (tt') = (11), (12). Sequences (21) and (22) lead to adoption pattern AB and consumer net benefits $w_1^{(tt')} = \bar{\gamma}_1^B + 2\theta x$, $w_2^{(tt')} = \bar{\gamma}_2^B + \theta x$, $tt' =$ (21), (22). All four adoption sequences are equilibria, but sequences (11) and (12) result in higher total consumer net benefits and adoption pattern BB, thereby resulting in an increase in market share for B. We can similarly show that if $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{BA}$, the equilibrium adoption sequences are (11) and (21), resulting in adoption pattern BB, which completes the proof. ■

Appendix B: Multiple Open Source Development Options

B.1 Example: Discrete Γ

Assume that open source developers, B, can chose from three discrete options: (i) no development, (ii) develop feature set $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$ at cost $\bar{c}$, or (iii) develop feature set $(\bar{\bar{\gamma}}_1^B, \bar{\bar{\gamma}}_2^B)$ at cost $\bar{\bar{c}}$: $\Gamma = \{(0,0), (\bar{\gamma}_1^B, \bar{\gamma}_2^B), (\bar{\bar{\gamma}}_1^B, \bar{\bar{\gamma}}_2^B)\}$. Now, instead of the binary decision of whether to develop a product or not, B must also decide on the product features if it decides to develop. As before, B maximizes its intrinsic motivation ϕ plus the weighted sum of consumer surplus across the two segments, $\phi + (\alpha w_1^{TT'} + \beta w_2^{TT'})x - C^B$.

Suppose the first open source option is such that $\bar{\gamma}_1^B > \gamma^* + \theta x - \frac{C^A(\gamma^*)}{x}$ and $\bar{\gamma}_2^B < \gamma^* - \theta x - \frac{C^A(\gamma^*)}{x}$, so that if A and B are incompatible, the adoption outcome is BA (Figure 2), and consumer net benefits are $w_1^{BA} = \bar{\gamma}_1^B + \theta x$ and $w_2^{BA} = \bar{\gamma}_2^B + 2\theta x$. Assume that $\phi + \alpha\bar{\gamma}_1^B + \beta\bar{\gamma}_2^B + (\alpha + 2\beta)\theta x - \bar{c} > 0$ so that it is worthwhile for open source developers to develop a product with feature benefits $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$. Now suppose there is another open source product option with better product features, $\bar{\bar{\gamma}}_1^B > \bar{\gamma}_1^B > \gamma^* + \theta x - \frac{C^A(\gamma^*)}{x}$ and $\bar{\bar{\gamma}}_2^B > \gamma^* + \theta x - \frac{C^A(\gamma^*)}{x} > \bar{\gamma}_2^B$, which costs $\bar{\bar{c}}$ to develop. If B develops a product with feature benefits $(\bar{\bar{\gamma}}_1^B, \bar{\bar{\gamma}}_2^B)$, the adoption outcome is BB and consumer net benefits are $w_1^{BB} = \bar{\gamma}_1^B + 2\theta x$ and $w_2^{BB} = \bar{\gamma}_2^B + 2\theta x$. Therefore, open source developers would choose to develop $(\bar{\bar{\gamma}}_1^B, \bar{\bar{\gamma}}_2^B)$ over $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$ if and only if $\alpha(\bar{\bar{\gamma}}_1^B - \bar{\gamma}_1^B) + \beta(\bar{\gamma}_2^B - \bar{\bar{\gamma}}_2^B) + \alpha\theta x \geq \bar{\bar{c}} - \bar{c}$. The first two terms on the left-hand-side are just the (weighted) increase in standalone value realized by the second option. The third term is the increase in network benefit as we shift from a split market (BA) to standardization on B. The difference in the development cost, $\bar{\bar{c}} - \bar{c}$, must be small enough to make the product feature improvements and increased network benefits worthwhile. Specifically, for low $\bar{\bar{c}}$, i.e., $\bar{\bar{c}} \leq \alpha(\bar{\bar{\gamma}}_1^B - \bar{\gamma}_1^B) + \beta(\bar{\gamma}_2^B - \bar{\bar{\gamma}}_2^B) + \alpha\theta x + \bar{c}$, product $(\bar{\bar{\gamma}}_1^B, \bar{\bar{\gamma}}_2^B)$ is optimal, otherwise, $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$ is optimal. For every product option that open source developers have, they make the same tradeoff of potential feature improvements and/or increased network size vs. cost of implementation.

B.2 Example: Continuous Γ

We show through the following numerical example that our qualitative results remain intact when Γ is continuous, namely that firm A can have a first-mover advantage (increasing profit and market share) by staggering the adoption of consumers and improving its standalone (feature) benefits. Let $x = 2$ and $\theta = 1$, so the network benefit from a user's own segment is $\theta x = 2$, and when both segments are compatible, $2\theta x = 4$. Let B's intrinsic motivation incentive be $\phi = \frac{2}{3}$. Let $\alpha = 1$ and $\beta = \frac{1}{3}$ so the open source developers are more interested in developing software for their own use (α) than as a career-motivated signalling mechanism (β). Finally, we assume for simplicity that $C^A(\gamma) = C^B(\gamma) = \gamma^2$.

B Available First Following the reasoning described in Section 5.1.2, B sets optimal standalone values $\tilde{\gamma}_1^B = \hat{\gamma} = 1$ and $\tilde{\gamma}_2^B = \hat{\hat{\gamma}} = \frac{1}{3}$ in period $1a$. In period $2a$, if firm A enters the market, its optimal standalone benefits are $\tilde{\gamma}_1^A = \tilde{\gamma}_2^A = \gamma^* = 1$, where $\gamma^* \equiv \{\gamma | x - \frac{\partial C^A(\gamma)}{\partial \gamma}\}$ as before. Although firm A would win both segments with this strategy, it's profit would be $(\tilde{\gamma}_1^A - \tilde{\gamma}_1^B + \tilde{\gamma}_2^A - \tilde{\gamma}_2^B)x - C^A(\gamma_1^A) - C^A(\gamma_2^A) = -\frac{2}{3}$. Since firm A cannot profitably enter the market, B wins both segments in period $2b$, resulting in adoption pattern BB, zero profit for firm A, and utility $\phi + (\alpha(\tilde{\gamma}_1^B + 2\theta x) + \beta(\tilde{\gamma}_2^B + 2\theta x))x - C^B(\tilde{\gamma}_1^B) - C^B(\tilde{\gamma}_2^B) = \frac{112}{9}$ for B. Note that intrinsic motivation alone would not have been a sufficient incentive for B to develop a product, i.e., $\phi < C^B(\tilde{\gamma}_1^B) + C^B(\tilde{\gamma}_2^B)$. In this case, B's optimal standalone benefits need to be sufficiently high compared to its development cost to make entry worthwhile.

B Moves First If both segments adopt A in period $1b$ (sequence `(11)`), firm A will not have a first-mover advantage (the intuition from Lemma 2 applies). If neither segment adopts in period $1b$ (sequence `(22)`), the analysis is very similar to the analysis for when B is available first resulting in B setting equilibrium feature benefits $\tilde{\gamma}_1^B = \hat{\gamma} = 1$ and $\tilde{\gamma}_2^B = \hat{\hat{\gamma}} = \frac{1}{3}$, winning both segments in period $2c$, making consumer net benefits $w_1^{BB} = \hat{\gamma} + 2\theta x = 5$ and $w_2^{BB} = \hat{\hat{\gamma}} + 2\theta x = \frac{13}{3}$, and firm A's profit equal to zero.

Consider now the case where segment 1 adopts A in period $1b$ (sequence `(12)`). Given the feature benefits of both products and firm A's period-2 segment-2 price (there is no segment-1 price since those consumers have already adopted A), consumers choose to adopt A or B in period $2c$ to maximize their net benefits. The necessary and sufficient conditions for each adoption pattern can be derived from the consumer payoff matrix shown in Figure 3(a). AA and BB are both equilibria if $-\theta x \leq \gamma_1^A - \gamma_1^B \leq \theta x$ and $\gamma_2^A - \gamma_2^B - \theta x \leq p_2^A(2) \leq \gamma_2^A - \gamma_2^B + \theta x$, in which case we choose the equilibrium with the higher total consumer net benefits, i.e., the equilibrium is AA if $p_2^A(2) \leq \gamma_1^A - \gamma_1^B + \gamma_2^A - \gamma_2^B$, and BB otherwise.

In period $2b$, firm A sets price $p_2^A(2)$ to maximize period 2 profit. Firm A can only make profit in period 2 if segment 2 adopts its product, i.e., the outcome is adoption pattern AA or BA. Since segment-1 consumers have to pay to adopt A in period $1b$, they will not do so if they plan to switch to B in period 2. Therefore, BA is not a feasible outcome, leaving AA as the only possible adoption outcome which could lead to a first-mover advantage for A. Firm A's second period profit when the adoption outcome is AA is $(\gamma_2^A - \gamma_2^B + \min\{\theta x, \gamma_1^A - \gamma_1^B\})x$,[5] resulting consumer net benefits $w_1^{AA} = \gamma_1^A + 2\theta x$ and $w_2^{AA} = \gamma_2^B + 2\theta x - \min\{\theta x, \gamma_1^A - \gamma_1^B\}$.

In period $2a$, anticipating A's pricing strategy in period $2b$, B sets its standalone benefits to maximize its objective function, $\phi + (\alpha w_1^{AA} + \beta w_2^{AA})x - C^B(\gamma_1^B) - C^B(\gamma_2^B)$. If $p_2^A(2) = \gamma_2^A - \gamma_2^B + \theta x$, then maximizing B's objective function with respect to γ_1^B and γ_2^B gives optimal feature benefits $\tilde{\gamma}_1^B = 0$ and $\tilde{\gamma}_2^B = \hat{\hat{\gamma}} = \frac{1}{3}$. If $p_2^A(2) = \gamma_2^A - \gamma_2^B + \gamma_1^A - \gamma_1^B$, then B's optimal feature benefits are $\tilde{\gamma}_1^B = \tilde{\gamma}_2^B = \hat{\hat{\gamma}} = \frac{1}{3}$.

In period $1b$, segment 1 consumers will adopt A if and only if they are at least as well off adopting A now as they would be if they waited for B's entry in period 2. If segment 1 waits until period 2, the sequence (22) game is played, resulting in adoption pattern BB and consumer net benefits $w_1^{BB} = \hat{\gamma} + 2\theta x$ and $w_2^{BB} = \hat{\hat{\gamma}} + 2\theta x$. Since the adoption outcome when firm A has a first-mover advantage is AA, in period $1a$, A must price so that segment-1 consumers are indifferent between net benefits $w_1^{AA} = \gamma_1^A + 2\theta x$ (if they adopt in period $1b$) and $w_1^{BB} = \hat{\gamma} + 2\theta x$ (if they adopt in period $2c$). This implies that $p_1^A(1) = \gamma_1^A - \hat{\gamma}$. Firm A sets segment-2 price sufficiently high so that segment-2 consumers delay adoption.

In period $1a$, firm A must also set its standalone values to maximize its profit over the two periods, which consists of revenue in the first period from segment 1, plus revenue in the second period from segment 2, minus the development cost, i.e., firm A's profit is $(\gamma_1^A - \hat{\gamma})x + (\gamma_2^A - \gamma_2^B) + \min\{\theta x, \gamma_1^A - \gamma_1^B\})x - C^A(\gamma_1^A) - C^A(\gamma_2^A)$. Maximizing profit with respect to γ_2^A gives optimal standalone benefit $\tilde{\gamma}_2^A = \gamma^* = \{\gamma | x - \frac{\partial C^A(\gamma)}{\partial \gamma} = 0\} = 1$. The optimal standalone benefit for segment 1 depends on B's reaction

[5]Firm A's profit is derived from the conditions for an AA equilibrium, i.e., $p_2^A(2) \leq \gamma_2^A - \gamma_2^B + \theta x$ and $p_2^A(2) \leq \gamma_1^A - \gamma_1^B + \gamma_2^A - \gamma_2^B$.

function. B wants to maximize consumer net benefits, subject to its own cost constraints. If firm A has a big enough cost advantage, it could force B out of the segment-2 market altogether. However, for our example, the best firm A can do is set optimal standalone benefit $\tilde{\gamma}_1^A = \gamma^{**} = \{\gamma | 2x - \frac{\partial C^A(\gamma)}{\partial \gamma} = 0\} = 2$, which leads to period-1 price $p_1^A(1) = \tilde{\gamma}_1^A - \hat{\gamma} = 1$. B responds in period $2a$ by setting $\tilde{\gamma}_1^B = \tilde{\gamma}_2^B = \hat{\hat{\gamma}} = \frac{1}{3}$, which leads period-2 price $p_2^A(2) = \tilde{\gamma}_1^A - \tilde{\gamma}_1^B + \tilde{\gamma}_2^A - \tilde{\gamma}_2^B = \frac{7}{3}$.

Firm A's profit over the two periods is $(\tilde{\gamma}_1^A - \hat{\gamma})x + (\tilde{\gamma}_2^A - \hat{\hat{\gamma}})x + (\tilde{\gamma}_1^A - \hat{\hat{\gamma}})x - C^A(\tilde{\gamma}_1^A) - C^A(\tilde{\gamma}_2^A) = \frac{5}{3}$. Therefore, by moving first, firm A can increase market share (from zero to winning both segments) and make positive profit. Note that consumer net benefits when A moves first are $w_1^{AA} = \tilde{\gamma}_1^A + 2\theta x - \tilde{p}_1^A = 5$ and $w_2^{AA} = \tilde{\gamma}_2^A + 2\theta x - \tilde{p}_2^A = \frac{8}{3}$. Compared to net benefits $w_1^{BB} = \hat{\gamma} + 2\theta x = 5$ and $w_2^{BB} = \hat{\hat{\gamma}} + 2\theta x = \frac{13}{3}$ when B is available first, the early adopters (segment 1) are equally well off, whereas the late adopters (segment 2) are exploited and end up worse off. B's utility when A moves first decreases to $\phi + (\alpha w_1^{AA} + \beta w_2^{AA})x - C^B(\tilde{\gamma}_1^B) - C^B(\tilde{\gamma}_2^B) = \frac{22}{3}$, compared to when B is available first. Note that in this case, $\phi > C^B(\tilde{\gamma}_1^B) + C^B(\tilde{\gamma}_2^B)$, so that B's intrinsic motivation incentive is higher than its development cost. Therefore, B would enter the market regardless of the eventual adoption outcome.

Appendix C: Asymmetric Segments

Suppose $x_1 < x_2 = x$. Then segment 1 generates less network value, and the standalone benefit it realizes contributes less to total network value, i.e., $\gamma x_1 < \gamma x$. These changes in value do not change how the adoption equilibria are derived, however, substitution of $x_1 < x_2 = x$ in the calculations will obviously change specific threshold values. One direct impact of a decrease in x_1 is that B is less likely to develop a product, as B's objective function, $\phi + \alpha w_1^{TT'} x_1 + \beta w_2^{TT'} x_2 - \bar{c}$, decreases with decreasing x_1. We can also think of x_1 and x_2 as weights in B's objective function. That is, if $x_1 < x_2$, the impact of segment 1's net benefit on B's decision to develop a product is less than segment 2's.

Decreasing x_1 can also change consumer behavior through changes in network value. A decrease in x_1 means that segment 2 consumers gain less from being compatible with segment 1. When A and B are incompatible and consumers have different preferences, they are trading off their desire for compatibility (to leverage network effects) with their standalone benefit preferences. The decrease in network value generated by segment 1 makes a split market outcome more likely (or equivalently, if $x = x_1 < x_2$, the increase in x_2 would make outcomes AA and BB more likely) because segment 2 consumers will tend to favor adopting their preferred technology. Figure 5 shows how the equilibrium adoption regions shift when $x_1 < x_2 = x$. Notice that the overall shape of the regions remains the same as before (indicating the structure of the analysis remains intact), however, the figure is skewed to reflect the asymmetry in segment sizes and the regions of split market outcome increase (indicated by the shaded areas).

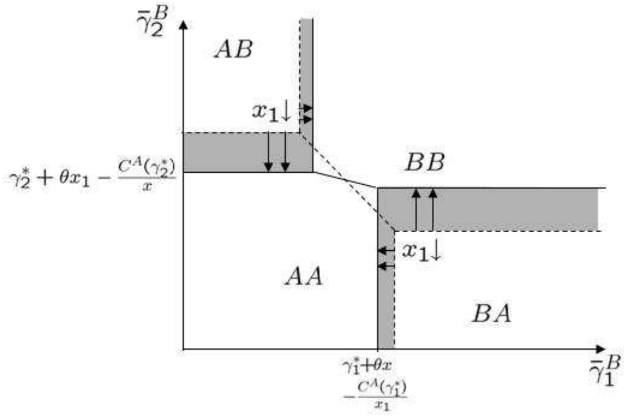

Figure 5: Adoption equilibria when B is available first and A and B are incompatible. TT' refers to the region where segment 1 adopts T (A or B) and segment 2 adopts T' (A or B). The shaded areas represent areas where the equilibrium outcome shifts to a split market when x_1 decreases. The dotted lines delineate equilibrium adoption regions when $x_1 = x_2 = x$.

Finally, asymmetry in segment sizes will change A's optimal prices and product features. Again, the analyses for determining A's prices and product features remain intact, however, the exact values will change to reflect the change in segment sizes. Specifically, A's profit function becomes $p_1^A x_1 + p_2^A x_2 - C^A(\gamma_1^A) - C^A(\gamma_2^A)$. Optimizing the profit function makes A's optimal feature benefit segment-specific, i.e., $\gamma_i^* = \{\gamma_i^A | x_i - \frac{\partial C^A(\gamma_i^A)}{\partial \gamma_i^A}\}$. A's pricing will change to reflect the change in network value that is generated by segment 1.

Appendix D: Positive Switching Cost

Consider now the case where there is a positive switching cost, $s > 0$. If firm A moves first, we expect its first-mover advantage to increase since it may be able to capture some of the switching cost as profit. Clearly, if A and B are compatible, positive switching costs will still not create a first-mover advantage: without network effects, it is optimal for consumers to just wait for the product they prefer and obtain it at the competitive price.

When A and B are incompatible and A moves first, positive switching costs affect the consumer payoff matrix in the second period when adoption is staggered (Figure 6). When neither segment adopts in period 1, the results from when B is available first in Lemma 3 hold. From the payoff matrices in Figure 6, we can determine the equilibrium period-2 consumer strategies for adoption sequences (12)

Seg i \ Seg j	Adopt A	Adopt B
Stay with A	$\gamma_i^A + 2\theta x$, $\gamma_j^A + 2\theta x - p_j^A(2)$	$\gamma_i^A + \theta x$, $\bar{\gamma}_j^B + \theta x$
Switch to B	$\bar{\gamma}_i^B + \theta x - s$, $\gamma_j^A + \theta x - p_j^A(2)$	$\bar{\gamma}_i^B + 2\theta x - s$, $\bar{\gamma}_j^B + 2\theta x$

(a) Only segment i adopts A in period 1.

Seg 1 \ Seg 2	Stay with A	Switch to B
Stay with A	$\gamma_1^A + 2\theta x$, $\gamma_2^A + 2\theta x$	$\gamma_1^A + \theta x$, $\bar{\gamma}_2^B + \theta x - s$
Switch to B	$\bar{\gamma}_1^B + \theta x - s$, $\gamma_2^A + \theta x$	$\bar{\gamma}_1^B + 2\theta x - s$, $\bar{\gamma}_2^B + 2\theta x - s$

(b) Both segments adopt A in period 1.

Figure 6: A moves first, period-2c consumer payoff matrix, A and B are incompatible, positive switching cost ($s > 0$).

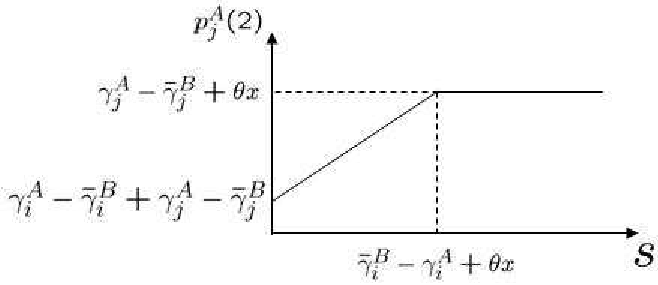

Figure 7: Segment-j, period-2 price as a function of switching cost, given adoption sequence (ij) and feature benefits (γ_1^A, γ_2^A), $(\bar{\gamma}_1^B, \bar{\gamma}_2^B)$.

and (21), given the feature benefits of A and B. Since the results from when B is available first are the same as when switching costs are zero, the period-1, segment-i price under sequence (ij) is the same as in Lemma 4, and Lemma 2 holds. The period-2, segment-j price under sequence (ij) is:

$$p_j^A(2) = \begin{cases} \gamma_j^A - \bar{\gamma}_j^B + \theta x, & \text{if } \bar{\gamma}_i^B \leq \gamma_i^A - \theta x + s \\ \gamma_i^A - \bar{\gamma}_i^B + \gamma_j^A - \bar{\gamma}_j^B, & \text{if } \bar{\gamma}_i^B > \gamma_i^A - \theta x + s \end{cases}. \tag{16}$$

Firm A's second period price, equation (16), increases with the switching cost (Figure 7). The increase in the second period price is the crux of firm A's increased first-mover advantage. If $s > 0$, A can capture part of the switching cost as profit from the late adopters. The analysis for first-mover advantage with positive switching cost is straightforward, however, in the interest of brevity, we will illustrate the impact of the switching cost by considering the case when switching costs are infinite. That is, when a consumer never switches once he adopts a technology.

Suppose A moves first. If firm A prices to stagger consumer adoption, it can capture the entire network value generated by its installed base. From Figure 7, if the switching cost, s, is infinite, the period-2, segment-j price under sequence (ij) is $p_j^A(2) = \gamma_j^A - \bar{\gamma}_j^B + \theta x$. The corresponding optimal segment-i feature benefit is γ^*, leading to maximum segment-i profit $(\gamma^* - \bar{\gamma}_i^B)x + \theta x^2 - C^A(\gamma^*)$. Regardless of technology B's segment-i feature benefit, $\bar{\gamma}_i^B$, firm A does not have to improve its product features above γ^*, the profit-maximizing level, because once segment-i has adopted, consumers cannot switch. *Therefore, switching cost enables firm A to capture network value without improving its product features.*

If B is available first, the period-2c consumer payoff matrices are as shown in Figure 6 except that A and B are switched. Since open source developers are interested in maximizing consumer surplus, the nature of their first-mover is not in the form of profit increase, rather, it is an increase in market share. Firm A's greed is actually the driver of B's first-mover advantage. Consumers know that once firm A enters in period 2, it will extract as much surplus as it can. If consumers adopt A, they end up with the same net benefit as they would have received by adopting B (firm A's strategy is to raise prices just until consumers are indifferent between A and B). Therefore, if B is available first, consumers who would have adopted A in period 2, are indifferent between adopting B in period 1 and waiting for A in period 2. This is summarized formally by the following Proposition.

Proposition 8 *Suppose consumers cannot switch. If $(\bar{\gamma}_1^B, \bar{\gamma}_2^B) \in D_{AB} \cup D_{BA}$, technology B gains market share if it is available first.*

When firm A enters in period 2, because there is no switching, it cannot win back the segment that adopted B in period 1. In the region where there would have been a split market equilibrium (AB or BA) when B is available first with zero switching cost, segment $i = 1$ or 2 that would have adopted A, is indifferent between adopting B early and waiting for A in period 2 (because firm A prices so that consumers are indifferent between adopting A or being compatible with segment j on B). However, segment-j consumers, who would have adopted B alone in the absence of switching costs, gain the network benefit by being compatible with segment i. The increase in *total* consumer net benefits by being compatible on technology B shifts the outcome from a split market equilibrium to compatibility on B, thereby increasing the market share of B. We can directly extend this analysis to finite, positive switching cost, leading to the same conclusion: B gains market share when switching costs are positive.