Abstract Number : 007-0461

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POMS 18th Annual Conference
Dallas, Texas, U.S.A.
May 4 to May 7, 2007
Models for effects of Manufacturing Marketing Environment on Manpower Planning

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Abstract

Shrinking margins and rapid changes in technology are forcing organisations world wide to realign their manufacturing as well as marketing practices. Many industries like computers, telecommunications, digital cameras, health care, auto mobile and auto component manufactures across the globe are experiencing reduced product life cycles, shortage of skilled work force as also demand volatility and market competition. We present a framework that balances significant trade-offs that help managers in crafting a strategy for the induction of relevant workforce such that the output is in line with the market demand. We considered various issues like operator skill levels, learning ability, wages, product life cycle issues, market demand, manufacturing process details etc. From the preliminary study it is found that meeting ramp-up in demand at different stages of product life cycle using mixed workforce is very complex and need scientific approaches, which are built on some simple analytical approaches.

Keywords: Skill sets, Manpower Planning, Production Planning

1. Introduction

The success of Japanese manufacturing has caused many others including American, Indian and many other manufacturers to search for ways in which it can gain competitive advantage. One of the key elements of Japanese manufacturing, generally known as Just In Time (JIT) production, is a Group Technology (GT)/ Cellular Manufacturing (CM), where machines are grouped into manufacturing cells to produce families of parts with similar shapes or processing requirements. The flexibility of a GT/CM system is derived mostly from its allocation of flexible workforce. A flexible workforce organized in groups can react quickly to temporarily unbalanced workloads. Worker cross-training that enables workers to perform a variety of tasks and relocate as the workload changes achieves this flexible workforce. American manufacturers have tried to identify and institutionalize work teams, which can jointly improve quality and flexibility. Standard job definitions and union work rules have been replaced with the team concept. Two examples of incorporating team concepts in major
industrial settings are Frito-Lay and General Motors. Research in flexible workforces has been mostly in dual resource constrained (DRC) job shop environments, where the number of workers is less than the number of machines, and both labor and machine resources must be available to process a job. Studies on a DRC shop scheduling have considered a variety of labor related decisions, such as labor flexibility, degree of centralization, size of work force, labor efficiency, labor assignment etc.

From the past studies, it is to be noted that flexible/cross trained worker force help in managing overloaded work centers, absenteeism, demand variations and other changes in the business. Worker cross training is especially important in light of the large number of firms that recognize the importance of worker cross-training and try to cross-train the workforce in order to adopt JIT principles. The system performance increases as the labor efficiency matrix changed from least efficient (no cross training) to most efficient (full cross training), but, the increase of system performance by moving from efficiency level of two to full efficiency (100% cross training) was not significant. The minimum introduction of worker cross-training showed the most significant improvement, and a subsequent increase in cross-training had a diminishing return. Instead of trying to train workers for every work center, workers should be trained so that they can work in at least two centers. This is an important suggestion considering the time and cost of worker cross-training and the complication due to union contracts. Further, worker cross training may have to focus on bottleneck work centers.

The degree of worker cross-training is represented by the labor efficiency matrix denoted A. Each element, \( a_{ij} \) in the matrix A represents the suitability of a worker \( i \) at the work center/process, \( j \), Where \( a_{ij} = 0 \), a worker \( i \) is unable to work at the work center/process \( j \). The labor efficiency matrix with \( a_{ij} =1 \) when \( i=j \), and \( e_{ij} =0 \) when \( i \neq j \) represents the machine limited job shop with no cross training (Park, 1991).
Firms such as computers, telecommunications equipment, cellular phones, digital cameras, electronic gadgets, auto electronics and many other products which follow short product life cycle (PLC) characteristics face a multitude of challenges on several fronts: exacting customer demands for a wide variety of products with short lead times, increasingly compressed product life cycles, and often, severe cost pressures (Fine, 1998). A common strategy used by managers in these firms to sustain competitive edge is frequent introduction of new products and this creates pressure on manufacturing to respond quickly to changing demand characteristics (Kurawarwala and Matsuo, 1996). Many firms including OEMs and component manufacturers like the one that is the focus of this study operate in a labour intensive environment and respond to changes in demand by multi-skilled workforce. The challenge for many managers is to determine how many additional workers to induct/train to satisfy dynamically varying market demand.

In order to meet the requirements at minimal cost, organizations use overtime capacity and/or skill flexibility of operators. Several other factors like manufacturing process with discrete sub-processes (e.g., assembly, inspection/test, rework) that require distinct skills complicate this problem. The workers’ skill set follows a nested structure i.e., highly skilled workers can perform complex and simple jobs while less skilled workers have limited proficiency. The processing requirements for each sub-process must be satisfied, given a limited inventory of skills of different levels of competence. Another related issue that needs to be incorporated in the decision is the impact of learning or the rate at which the workers can learn to perform jobs to full potential. Managers also need to assess the value of different cross training strategies e.g., limited versus complete cross training. The objective is to study the impact of cost of induction/training, overtime premium, worker flexibility and learning rate on various
costs. This helps managers in defining a strategy for allocation of workers in a complex assembly environment. Our numerical study indicates that both the firm’s cost performance and the number of workers need to induct/train are significantly affected by key parameters such as cost of induction/training, overtime premium cost and flexibility configuration adopted. We highlight the impact of cross training strategies and emphasize the overall value of the methodology through managerial insights drawn.

The rest of this paper is organized as follows. In Section 2 we describe the details of the manufacturing process and uniqueness of the current study. Section 3, presents methodology and analysis of the key issues of operators induction/training, learning rates, effect of flexibility on induction and other costs. Finally in Section 4, we present our conclusions and directions for future work.

2. Motivation, problem context and Operator skills

The review of literature emphasizes the need for decision models pertaining to three types of decisions – planning, scheduling and allocation (Campbell 1999). Often other complicating issues such as effect of learning and workforce skill flexibility needs to be incorporated into the decisions. An important need identified in literature is that of integrating the three types of models. In our earlier paper (Rohit Bhatnagar and Venkat, 2004), we have focused on the planning problem for minimizing the labour costs with single flexibility configuration. This, being one of the important predicaments in many manufacturing industries today, provides a substantial scope for improvement. This depth issue therefore deals with developing a decision support system to evaluate the induction/training rate of different workforce considering different skills for workforce primarily based on varying demand and learning curve for different product types. This research was motivated by a project carried out by the author at a computer assembly firm in Singapore and couple of auto component
manufacturers in India. The firm’s manpower strategy is to have a workforce with multiple skills such that the demand at different stages of the life cycle can be met. The general operations include manufacturing lines that can operate in parallel. The manufacturing process comprises six main stages as shown in Figure 1.

![Figure 1: Manufacturing Process](image)

Customer orders are released into assembly according to the daily production plan. After first stage of assembly, the units are tested at testing stage before they enter into subsequent assembly stage. After second stage of assembly the subassemblies are tested again and depending on the status of the products are dispatched or reworked. The items that successfully complete the second test are sent for packing and dispatch while the others fail at any of the inspection/testing stages must undergo rework. The output from the packing stage determines the final throughput of the line. Though there are six stages, the manufacturing process essentially requires three main types of competence – assembly, inspection/test and rework. The output at each stage depends on the number of workers allocated to that stage and the process yield. The theoretical output at processing stage j, for model m is given by the number of workers at that stage times the standard output rate per worker. The yield depends on the quality of components and the accuracy of the assembly process. The defects may be detected at two stages of inspection/testing i.e., Testing 1 and Testing 2. The rejected units are sent to the rework stage where highly skilled operators or technicians repair the units and send them back to the inspection/testing process. The workers go through required training on the job training. An operator can reach the standard output rate on a given
workstation within a certain period based upon learning characteristics described as standard learning curve given in Table 4 (Nahmias, 2001).

3. Workforce skills and flexibility configurations

The workers in the background firms, possessing four different skills (U, V, W and Z) depending on their competence in one or more of the manufacturing processes as shown in figure 2. This is considered as basic configuration where assembly operations are done by all the categories and other operations done by two categories further U can only do assembly operations and Z can do all the three operations. Workforce competency is given by means of a binary relation \( a_{ij} \) and is shown in Table 1. An \( a_{ij} \) value of 1 indicates the competence of worker in category i in process j and zero denotes incompetence of the worker.

<table>
<thead>
<tr>
<th>Skill Type</th>
<th>Process Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>Assembly</td>
</tr>
<tr>
<td>V</td>
<td>Testing</td>
</tr>
<tr>
<td>W</td>
<td>Rework</td>
</tr>
<tr>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Workforce Skill-Process Relation

Other four different flexibility configurations have been derived from basic flexibility with valid assumptions and are given in Table 2. Apart from basic (current practice) configuration (A1) and complete flexibility (configuration A5) three more intermediate configurations have been considered to study the impact on total cost, induction/training etc. In configuration A2, it is assumed that each skill category (U, V, W) performs one operation except Z category.
specialised in all the three operations. This configuration results in two skill categories eligible for each process. Configuration A3 is assumed such that each skill category can perform two processes and similarly under configuration A4 each skill category can do only one job which is least flexible option among all other configurations. The summary of these five configurations is given in Table 3.

Table 2: Workforce additional Flexibility Configurations \( (A^i_{ij}) \)

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Flexibility Configuration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
<td>Current practice- Basic configuration, Assembly operation done by all skill categories, Z does all operations, V and W inspection and repair respectively apart from assembly process</td>
</tr>
<tr>
<td>2</td>
<td>A2</td>
<td>U, V, W skill category specialized in one process and Z in all the three. This results in two eligible skill categories for each process</td>
</tr>
<tr>
<td>3</td>
<td>A3</td>
<td>Each skill category specialized in two processes uniformly</td>
</tr>
<tr>
<td>4</td>
<td>A4</td>
<td>Each skill category specialized in only one process</td>
</tr>
<tr>
<td>5</td>
<td>A5</td>
<td>Each skill category can perform all the three processes</td>
</tr>
</tbody>
</table>

Table 3: Summary of Workforce skill configurations
Whenever there is a ramp up in market demand/production volume, capacity is increased by means of overtime and/or induction of suitable workforce. Contingent/casual workers are more cost effective as they are paid less compared to permanent workers. However, they have a limited set of skills and can only be engaged in specific processes and can be trained as required. In the firms we studied, the use of contingent/casual workers was limited to the assembly process. Moreover, contingent workers being new to the manufacturing process, require a “warm up” period before they reach their maximum potential output follows a learning curve as shown in Table 4 wherein A denotes fast learning and B denotes slow learning and are adopted from Nahmias (2001). These are close to the current practices of the firms under study. Other categories of workforce also follow similar pattern under new skill categories.

<table>
<thead>
<tr>
<th>Days since induction/start</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>….</th>
<th>18</th>
<th>….</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (fast) Learning</td>
<td>50</td>
<td>83</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>….</td>
<td>100</td>
<td>….</td>
<td>100</td>
</tr>
<tr>
<td>B (slow) Learning</td>
<td>50</td>
<td>59</td>
<td>65</td>
<td>69</td>
<td>73</td>
<td>….</td>
<td>100</td>
<td>….</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4: Workforce Learning Rates

One option for a meeting the ramp in market demand/production is to increase the pool of workers/skill sets. However, this results in excess capacity and higher labour costs during lean demand periods. Contingent labor provides more economic and flexible capacity. Our discussions with production managers threw up some key issues that were confronting them. How should workers of various skill levels be allocated to different production processes to achieve maximum throughput? How should permanent and contingent workers be allocated to different shifts and how much overtime was appropriate? At what rate should the contingent workers (having a specific learning rate) be inducted to minimize labor cost while achieving the desired ramp up? What is the value of cross-training of workforce in handling a broader portfolio of processes? These issues have been incorporated in our earlier study (Rohit Bhatnagar and Venkat 2004). The main focus of this study is to investigate different
flexibility configurations and their impact on various costs and induction/training of workers in meeting the demand requirement.

Wage differentials exist for permanent/contingent workforce, regular/overtime work as well as for different shifts and vary between 100 and 206.25%. Due to confidentiality reasons, the least cost shift is taken as 100% and highest cost shift is three under overtime for permanent category. The typical demand at aggregate level (for four models) is shown in figure 3, which follows a typical product life cycle pattern. The planning period is considered as one month (30 days). Process yield is assumed as constant at 95% and OT capacity as 30% of regular time capacity. The key variables considered in this study and their levels are shown in Table 5.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Levels</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility</td>
<td>5</td>
<td>Basic, 100% flexible and other three cases</td>
</tr>
<tr>
<td>Cost of Induction</td>
<td>3</td>
<td>Cost of inducting each worker(^1)</td>
</tr>
<tr>
<td>Learning Curve</td>
<td>2</td>
<td>Fast and Slow learning rates(^2)</td>
</tr>
<tr>
<td>Overtime Cost</td>
<td>3</td>
<td>Premium of 25%, 50%, and 100% over regular salary(^3)</td>
</tr>
</tbody>
</table>

\(^1\)Cost of induction/training worker is equivalent of 1, 2, or 3 weeks salary respectively.

\(^2\)New workers/skills assumed to reach 100% capability in 3 and 18 days respectively.

\(^3\)Levels of overtime cost and amount reflected the typical firm practice and variants around it

Table 5: Experimental Variables

Figure 3: Demand Characteristics
4. **Methodology and Analysis of Results**

The problem is modeled as an LPP (see the appendix) and solved using PC version of LINGO 7.0 and observed that the maximum CPU time was less than 5 Secs. For 30 day planning horizon, the above model represented a medium sized LP problem comprising approximately 17,700 variables and 9400 constraints. We present a brief analysis of results in respect of total cost, induction cost, OT and RT cost and induction of contingent workers at different OT premium and induction cost at two learning rates in respect of all the five flexibility configurations (given in Table 1 and Table 2). There are totally 90 observations resulted for the parameters listed in Table 5. We believe that these analyses provide useful managerial insights for key decision problems outlined in this study. They also enrich the decision making environment by providing useful framework for making important tradesoffs. Total cost is one of the important measures that reflect the practices. In the proposed model, cost comprised several components – cost of regular, overtime salary and idle time cost for permanent and contingent workers, induction cost for contingent workers.

The cost figures are reported as a proportion of the least cost obtained in all experiments (the least cost outcome is defined as 1.00). The variation is around 40%. It is to be noted that the flexibility configurations A1, A3 and A5 are comparable and A4 is least preferred configuration from total cost criteria followed by A2. The reason is that under A2 and A4 more number of contingent workers have been inducted due to higher amount of assembly work content which is less shared by permanent/skilled workers. The cost difference between least and complete flexible configuration is around 9% at aggregate level. Further, the benefit of worker flexibility increases as cost of induction increases from C1 to C3. Configuration A3, wherein for each process two skill categories eligible, is emerged as an alternative to current practice (A1) and complete flexible configuration (A5). It shows that
there is no need for 100% flexibility and further it may create lot more problems at scheduling level due to many alternatives and may lead to loss of morale and accountability. This observation is in agreement with previous researchers.

<table>
<thead>
<tr>
<th>OT Cost Premium</th>
<th>Flexibility Configuration / Learning</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% (HR1)</td>
<td>C1 1.00</td>
<td>1.01</td>
<td>1.04</td>
<td>1.04</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>C2 1.13</td>
<td>1.13</td>
<td>1.17</td>
<td>1.18</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>C3 1.23</td>
<td>1.24</td>
<td>1.29</td>
<td>1.30</td>
<td>1.23</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Avg 1.12</td>
<td>1.13</td>
<td>1.17</td>
<td>1.17</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>50% (HR2)</td>
<td>C1 1.01</td>
<td>1.02</td>
<td>1.04</td>
<td>1.05</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>C2 1.16</td>
<td>1.16</td>
<td>1.20</td>
<td>1.20</td>
<td>1.15</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>C3 1.27</td>
<td>1.28</td>
<td>1.32</td>
<td>1.33</td>
<td>1.27</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>Avg 1.14</td>
<td>1.15</td>
<td>1.19</td>
<td>1.20</td>
<td>1.14</td>
<td>1.15</td>
</tr>
<tr>
<td>75% (HR3)</td>
<td>C1 1.01</td>
<td>1.02</td>
<td>1.05</td>
<td>1.06</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>C2 1.17</td>
<td>1.19</td>
<td>1.22</td>
<td>1.23</td>
<td>1.17</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>C3 1.30</td>
<td>1.32</td>
<td>1.35</td>
<td>1.36</td>
<td>1.30</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Avg 1.16</td>
<td>1.17</td>
<td>1.20</td>
<td>1.22</td>
<td>1.16</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>Min 1.00</td>
<td>1.01</td>
<td>1.04</td>
<td>1.04</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>Max 1.30</td>
<td>1.32</td>
<td>1.35</td>
<td>1.36</td>
<td>1.30</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Avg 1.14</td>
<td>1.15</td>
<td>1.18</td>
<td>1.19</td>
<td>1.14</td>
<td>1.15</td>
</tr>
</tbody>
</table>

**Table 6: Relative Total Cost for Different flexibility configurations and OT premium at 30% OT Capacity**

The variation in relative total induction cost (shown in figure 5) is more than 500% for the five configurations considered. At lowest cost of induction (C1) the impact of flexibility on induction cost is almost the same at both learning rates across all the five configurations. The benefit of worker flexibility increases as cost of induction increases and the benefit is maximum at highest cost of induction. At highest cost of induction the suitable flexibility configuration would be in the order of A3, A1, A5, A2 and A4.
Figure 4: Relative Total cost at different Cost of Induction and flexibility configurations at fast and slow learning

Figure 5: Relative Total Induction Cost at different CI and flexibility configurations at fast and slow learning

The impact of overtime premium at different flexibility configurations are shown in figure 6. At lowest cost of induction (C1) under low OT premium cost (HR1) all the five flexibility configurations are almost the same in terms of relative total cost. Further, the difference due to learning is less than 3%. When OT premium changes from HR1 to HR2 and HR3 (HR1<HR2<HR3) the benefit of flexibility also increases. What is the managerial significance of this finding? With a globally disbursed manufacturing base (as is the case with the firm under study), worker induction at different locations is subject to local recruitment policies, practices and agency contracts. Since the relative induction costs are location dependent, managers with a global manufacturing coordination responsibility will find the above analysis useful in determining the relative cost impact of allocating different levels of production to different facilities.
Flexibility and capacity (as represented in this case by contingent worker induction) can be seen as substitutes. As Slomp and Molleman (2002) observe, a flexible workforce leads to higher utilization, lower cost, and is an effective tool for handling uncertainty. Figure 7 shows that for the firm under study, a more flexible workforce will lead to lower overall cost as well as induction of fewer number of contingent workers. Moreover, a medium increase in flexibility yields full benefits in terms of lower cost and worker induction. It is therefore not necessary to have complete flexibility (100%) in order to achieve full cost benefits. This finding supports results obtained previously in other contexts. It is also found that one of the proposed configurations (A3) is marginally better than the current practice (A1). Wherein, under A3 each skill category performs two different processes unlike in A1 where U category does only one type of job and Z category specialised in all the three processes.

Figure 6: Relative Total Induction Cost at different OT premium and flexibility configurations at fast and slow learning
Figure 7: Total Avg Cost and Induction cost Versus Flexibility

5. Conclusions

In this brief study, we proposed an LP model to examine the decision problem of selecting an optimal cost strategy suitable for manufacturing firms that uses both permanent and contingent workers. The manufacturing process consists of discrete sub-processes and workforce with four different nested skills. Other factors considered include worker learning, worker (skills) flexibility configurations, cost of induction, OT cost premium, market demand and product life cycle characteristics. The model represents substantive complexities of the firm’s production system. Our results throw up useful insights relating to manpower (with different skill sets) planning in a complex manufacturing system.

First, our analysis suggests that there is a significant relative total cost difference around 40% and the difference in relative induction cost is more than 500% across five flexible configurations studied. This information is crucial for managers from the perspective of cost control. The most important factors are the cost of inducting/training of workers, total cost, and worker flexibility/cross training. Managers with a global manufacturing coordination responsibility will find these results useful in planning and allocation of production to different facilities. Second, our results indicate that the optimal number of workers inducted/trained depends on a complex interaction between the induction cost, overtime-cost premium
and flexibility configuration. At higher costs of induction, the induction of contingent workers is less and hence higher overtime utilization across different processes. This situation demands higher degree of worker flexibility. At lower costs of induction, the overtime-cost premium plays an important role in determining the number of workers inducted/trained. Importantly, at high cost of induction, our model suggests that it is beneficial to induct workers as early as possible. The third important finding relates to worker flexibility achieved by cross training workers to perform a larger skill set. Our results supported the previously reported results that enhanced flexibility leads to reduced costs (and also lower contingent worker induction in our case). Importantly, partial flexibility was sufficient to achieve the full range of benefits. Our model adds value to managerial decision-making by providing analytical insights into the optimal allocation of different categories of workers to different shifts. Among the factors evaluated except training the impact of other factors is significant. The benefit of worker flexibility increases with increase in induction cost.

One of the limitations of our model is that it deals with the higher level-planning problem for induction/training of different skill category of workers. It therefore leaves the detailed scheduling issues unaddressed. Another issue is that the scenarios we have considered are predominantly deterministic. Future research needs to incorporate these perspectives. Several interesting extensions from this work can be addressed in future research. We have primarily addressed the decision problem in terms of cost based performance measures. Several variations in the induction policy need to be addressed. As suggested by Milner and Pincker (2001), worker induction in many contexts needs to be planned at different epochs of time (e.g., before and after demand revelation). For flexibility related aspects, training time and costs as well as several levels of worker competence are other promising research directions.
Acknowledgements

The author sincerely expresses his thanks to Singapore MIT Alliance (SMA) Program, Singapore for sponsoring the part of this study. He also acknowledges the support rendered by local auto component manufacturers and Indian Institute of Management for their support.

References


Appendix 1

Manpower Planning Model (MPM)

Index
I worker skills category \{1 = U, 2 = V, 3 = W, 4 = Z\}
J processing stages \{1 = Assembly, 2 = Inspection/Testing, 3 = Rework\}
K shifts \{k = 1, 2, 3\}
L production lines \{l = 1, 2, 3, 4\}
M products \{m = 1, 2, 3, 4\}

Parameters

$D^m_t$ demand for model $m$ on day $t$

$y^m_t$ yield for model $m$ on day $t$

$D^m_{jt}$ workload for model $m$ at processing stage $j$ on day $t$

- $D^m_t$ for $j=1$ (assembly)
- $D^m_t \left\{ 2 - y^m_t \right\}$ for $j=2$ (inspection/testing)
- $D^m_t \left\{ 1 - y^m_t \right\}$ for $j=3$ (rework)

$W^i_p$ number of permanent workers of skills category $i$

$a_{ij}$ = 1 if workers of skill category $i$ can work at processing stage $j$

= 0 otherwise

$U^m_{j}$ standard output rate for model $m$ at processing stage $j$

$L^i_t$ learning index for workers $t$ days after starting work (Table 4)

$C^i$ cost of inducting each contingent worker

$\alpha_k$ regular hourly rate for permanent workers during shift $k$

$\alpha'_k$ overtime hourly rate for permanent workers during shift $k$

$\beta_k$ regular hourly rate for contingent workers during shift $k$

$\beta'_k$ overtime hourly rate for contingent workers during shift $k$
Decision Variables

\[ X_{j}(l,k,t) \] number of permanent workers allocated to stage \( j \), on line \( l \), shift \( k \), day \( t \)

\[ Y_{j}(l,k,t) \] number of contingent workers allocated to stage \( j \), on line \( l \), shift \( k \), day \( t \)

\[ WP_{ikt}^{i} \] number of permanent workers of skill category \( i \) working in shift \( k \) on day \( t \)

\[ X_{j}^{'}(l,k,t) \] total number of overtime hours done by permanent workers at stage \( j \), on line \( l \), shift \( k \), day \( t \)

\[ Y_{j}^{'}(l,k,t) \] total number of overtime hours done by contingent workers at stage \( j \), on line \( l \), shift \( k \), day \( t \)

\[ X_{j}^{i}(l,k,t) \] number of permanent workers from skills category \( j \), allocated to work at stage \( j \), on line \( l \), shift \( k \), day \( t \)

\[ Y_{t}^{I} \] number of contingent workers inducted on day \( t \) of ramp up phase

Minimize \( \text{Total Cost} \)

\[
\sum_{j \in J} \sum_{l \in L} \sum_{k \in K} \sum_{t \in T} \left\{ X_{j}(l,k,t) * \alpha_{k} + X_{j}^{'}(l,k,t) * \alpha_{k}^{'} + Y_{j}(l,k,t) * \beta_{k} + Y_{j}^{'}(l,k,t) * \beta_{k}^{'} \right\}
+ \sum_{t \in T} Y_{t}^{I} * CI + \sum_{i \in I} \sum_{t \in T} \left\{ WP_{ikt}^{i} - \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} X_{j}^{i}(l,k,t) * a_{ij} \right\} * \alpha_{1}

\]

subject to

\[ X_{j}(l,k,t) - \sum_{i \in I} X_{j}^{i}(l,k,t) * a_{ij} = 0 \quad \forall \ j, l, k, t \quad \text{----(1)} \]

\[ \sum_{j \in J} \sum_{l \in L} X_{j}^{i}(l,k,t) * a_{ij} = WP_{ikt}^{i} \quad \forall \ i, k, t \quad \text{----(2)} \]

\[ \sum_{k \in K} WP_{ikt}^{i} \leq WP^{i} \quad \forall \ i, t \quad \text{----(3)} \]

\[ WP_{ikt}^{i} + WP_{ikt+1}^{i} \leq WP^{i} \quad \forall \ i, t, k = 1 \quad \text{----(4)} \]

\[ \sum_{j \in J} \sum_{l \in L} \left\{ X_{j}(l,k,t) + Y_{j}(l,k,t) \right\} + \sum_{j \in J} X_{j}^{'}(l,k,t) + Y_{j}^{'}(l,k,t) \} \geq \sum_{m \in M} \left\{ \sum_{j \in J} \sum_{l \in L} D_{j}^{m} \right\} \quad \forall \ j, t \quad \text{----(5)} \]
\[
\sum_{j \in J} \sum_{l \in L} X^i_j(l,k,t) - \sum_{l \in L} \sum_{i \in I} X^i_j(l,k,t) \cdot a_{ij} = 0 \quad \forall \ j, k, t \quad ----- (6)
\]

\[
\sum_{j \in J} \sum_{l \in L} X^i_j(l,k,t) \cdot a_{ij} - WP^i_{k-1,t} \cdot 4 \cdot 0.5 \leq 0 \quad \forall \ i, k, t \quad ----- (7)
\]

\[
\sum_{l \in L} Y^i_j(l,k,t) - \sum_{l \in L} Y^i_j(l,k-1,t) \cdot 4 \cdot 0.5 \leq 0 \quad \forall \ k, t, j = A \quad ----- (8)
\]

\[
X^i_j(l,k,t) + Y^i_j(l,k,t) + \{X^i_j(l,k,t) + Y^i_j(l,k,t)\} / 8 \leq \mathbb{R}_j \quad \forall \ j, l, k, t \quad ----- (9)
\]

\[
\sum_{l \in L} \sum_{k \in K} Y^i_j(l,k,t) - \sum_{p=1}^{p=t} LI_{t-p+1} \cdot Y^i_j = 0 \quad \forall \ j = A, t \quad ----- (10)
\]

Constraint 1 defines the number of permanent workers from skill set \( i \) allocated to processing stage \( j \), at each line \( l \), for each of the three shifts on all the days, where \( a_{ij} \) represents the feasibility of such an allocation (as shown in Table 1). Constraints 2 and 3 together ensure that the total number of permanent workers allocated from each skill set \( i \) to all feasible processing stages \( j \), lines \( l \), and shifts \( k \), on any given day, should not exceed the number of workers available for \( i \) (i.e. \( WP^i \)). Constraint 4 requires that any permanent worker employed in shift 3 on a given day be not allocated to shift 1 on the following day. Constraint 5 specifies that sufficient permanent and contingent workers must be deployed and overtime utilized if necessary on each day, so that the workload for all models is fulfilled at all processing stages. The right hand side of Constraint 5 defines the total number of worker-hours required to fulfill daily demand at each processing stage \( j \). This depends on the daily workload and the standard hourly output at each processing stage. Constraints 6 and 7 link the amount of overtime performed by permanent workers at each processing stage in any shift, to the number of permanent workers of appropriate skill category who were working in the previous shift. Both constraints are necessary in order to ensure that overtime limits are not exceeded. The overall limit for overtime work is 4 hours per worker for at most 50 percent of the workers in the previous shift. Constraint 8 defines limits on the amount of
overtime that can be performed by contingent workers. Constraint 9 specifies an upper limit on the number of equivalent workers (permanent and/or contingent) who can man a given production line in a given shift. Finally, Constraint 10 incorporates the characteristics of the learning curves for contingent workers. The objective function seeks to minimize the total cost comprising regular and overtime cost for permanent and contingent workers, the induction cost for the contingent workers and the idle time cost for the unutilized permanent workers, who must anyway be paid their wages. Idle time costs for permanent workers are accounted at the rate of the wages for shift 1.