Appendix

Proof of Lemma 1: When Condition 5 holds, Supplier 1's optimal spot market lot size is given by: Min{ $(Q_1\text{-}H)^+$, $(B_H\text{-}Q_2\text{-}s)^+/2$} when D=H, and by Min{ $(Q_1\text{-}L)$, $(B_L\text{-}Q_2\text{-}s)^+/2$} when D=L. The proof of the Lemma is based on showing that when β>-2, in equilibrium the production quantities satisfy: $(Q_1 - H)^+ \le 0.5(B_H - Q_2 - s)^+$. In equilibrium it must be that $Q_1 \le Max\left(H + 0.5(B_H - Q_2 - s),\ L + 0.5(B_L - Q_2 - s)\right)$, or else Supplier 1 deviates and produces less. In addition, when β>-2 we also have that $Max\left(H + 0.5(B_H - Q_2 - s),\ L + 0.5(B_L - Q_2 - s)\right) = H + 0.5(B_H - Q_2 - s)$. More details are available from authors by request.

Derivation of Table 2: Using Equations 6 and 11, we first write Supplier 1's profit as a function of the production quantities for the four cases, depending on whether $Q_1 \ge H$ and whether $Q_1 < L + 0.5(B_L - Q_2 - s)$. We only consider equilibriums in which both suppliers participate in the spot market with a positive probability, and in which the expected spot price is positive for every realization of the contracted demand. We search for production quantity equilibrium in each of the above four cases. We use Equation 10 to determine Supplier 2's best response function. In addition, we use the following relationships: $B_H = B + \beta(1-\alpha)(H - L)$, $B_L = B - \beta\alpha(H - L)$ and $B_H - B_L = \beta(H - L)$. Details are available from authors by request.

Proof of Propositions 1 and 2: We first examine the case in which supplier 2 does not observes D, and then the case in which he observes D.

(*a*) Non-transparent markets: in not-transparent markets $q_2[L]$= $q_2[H]$=Q_2, $q_1[L]$=Min[$(B_L\text{-}Q_2\text{-s})^+/2$, $Q_1\text{-}L$], and for L<Q_1<H we have:

$$\pi(Q_1) = \alpha(wQ_1 - k(H - Q_1)) + (1-\alpha)(wL + q_1[L](B_L - q_1[L] - Q_2) + s(Q_1 - L - q_1[L])) - cQ_1 .$$

We show in the derivation of Table 2 that there is no liquidation equilibrium in which $q_1[L]=(B_L-Q_2\text{-s})/2$. Thus, it is now sufficient to show that there is no equilibrium in which $q_1[L]=0$. Details are available from authors by request.

(*b*) Transparent markets : Given Q_2 and D, the optimal lot size for Supplier 2 when $L<Q_1<H$ is: $q_2[H]=\text{Min}[Q_2, 0.5(B_H- s)^+]$ and $q_2[L]=\text{Min}[Q_2, 0.5(B_L-q_1[L]-s)^+]$. When β is positive: $B_H> B_L$ and thus $B_H-s > B_L-q_1[L]-s$ and $q_2[H] \geq q_2[L]$ in an equilibrium with $q_1[H]=0$. Therefore, with positive correlation it must be that $q_2[H]= Q_2$ and $q_2[L] \leq Q_2$. When β is negative: $B_H < B_L$ but it might be that $B_H\text{-s} > B_L-q_1[L]-s$ and thus we can not determine yet whether $q_2[H]= Q_2$ and $q_2[L] \leq Q_2$ or $q_2[L]= Q_2$ and $q_2[H] \leq Q_2$.

- Looking for an Equilibrium in which $L<Q_1<H$, $q_1[H]=0$, $q_2[L]=q_2[H]=Q_2$ and $q_1[L]=Q_1-L$, we get the same production quantities as in the ILE. It is easy to show that there can not be an Equilibrium with $L<Q_1<H$, $q_1[H]=0$, $q_2[L]=q_2[H]=Q_2$ and $q_1[L]<Q_1-L$ (same as the proof when Supplier 2 does not observe D).
- We can show (details are available from authors by request) that there can not be an Equilibrium in which $\boldsymbol{q_2[L]= 0.5(B_L-q_1[L]\text{-s})^+ < Q_2}$**,** $\boldsymbol{q_2[H]=Q_2 >0}$, $\boldsymbol{L<Q_1 <H}$ **and** $\boldsymbol{q_1[H]=0}$ **(regardless of what is** $\boldsymbol{q_1[L]}$**)** . For Supplier 1 to produce less than H it must be that $\alpha(w+k)+(1-\alpha)s < c$. In addition it must be that $B_H-Q_2< (w+k)$ or else Supplier 1 deviates and deliver less than Q_1 units to the contracting customers when $D=H$. Hence, $\alpha(B_H-Q_2)+(1-\alpha)s < c$. But if that is true then Q_2 can not be an equilibrium production quantity such that Supplier 2 puts Q_2 on the market when $D=H$ and less than Q_2 units on the market when $D=L$ (i.e. he sells some units at the fixed price, s, when $D=L$). Supplier 2 would be better off deviating and producing less because the expected "gain" for the last unit produced given by $\alpha(B_H-Q_2)+(1-\alpha)s$ is smaller than its production cost, c, not to mention the reduction in price received for the

Q_2-1 units sold on the market when $D=H$. This is true with both negative and positive correlation and therefore, with negative correlation it must be that $q_2[L]= Q_2$ and $q_2[H] \leq Q_2$ and with positive correlation only the ILE, in which $q_2[L]= q_2[H] = Q_2$, can be feasible.

- If there is negative correlation between demands, there can be an equilibrium with $\boldsymbol{L<Q_1<H}$, $\boldsymbol{q_2[H] < Q_2}$, $\boldsymbol{q_2[L]= Q_2}$ and q_1[L]=Q_1-L, as given in Proposition 2. As before, we can show that there is no equilibrium in which $L<Q_1<H$, Supplier 1 is loyal, and $q_1[L]=B_L-q_2[L]-s <Q_1-L$ when $q_2[L]=Q2$ and $q_2[H]<Q_2$.

Proof of Proposition 3: $\partial Q_2 / \partial\alpha = (2B + c - 3(k+w) + \beta((-\alpha^2 - 6\alpha + 3)(H-L)))/(3+\alpha)^2 < 0$ if and only if $2B + c < 3(k+w) - \beta(3 - \alpha^2 - 6\alpha)(H-L)$. But, for the ILE to exist it must be that $2B + c < 3(k+w) - 3\beta(1-\alpha)(H-L)$ (see Table 2). Because $(3 - \alpha^2 - 6\alpha) \leq 3(1-\alpha)$ for every $\alpha\in[0,1]$ we conclude that for $\beta \geq 0$ we have $\partial Q_2 / \partial\alpha < 0$. When $\beta < 0$, $\partial Q_2 / \partial\alpha < 0$ if and only if $2B + c < 3(k+w) - \beta(3 - \alpha^2 - 6\alpha)(H-L)$ because this constraint is more binding.

$\pi_2 = (Q_2)^2$ and $\partial\pi_2 / \partial\alpha = 2Q_2(\partial Q_2 / \partial\alpha) < 0$ if and only if $\partial Q_2 / \partial\alpha < 0$. Hence, Proposition 3 holds for any $\beta > 0$, which satisfies the conditions for the ILE listed in Table 2, but only when $2B + c < 3(k+w) - \beta(3 - \alpha^2 - 6\alpha)(H-L)$ for $\beta < 0$. ■

Proof of Proposition 4: Supplier 1's expected profit increase from his access to the spot market is higher than Supplier 2's expected profit from the spot market iff $(1-\alpha)(Q_1 - L)^2 - Q_2^2 > 0$ that is iff $(1-\alpha)\left(Q_1^0 - \beta 2(E[D]-L)/(3+\alpha) - L\right)^2 - \left(Q_2^0 + \beta(1-\alpha)(E[D]-L)/(3+\alpha)\right)^2 > 0$ Which holds for

$$B \in \begin{pmatrix} k + w - \beta(H - E[D]) - \dfrac{\sqrt{1-\alpha}}{1-\alpha}\left(k + w - c - \beta(H - E[D])\right), \\ k + w - \beta(H - E[D]) + \dfrac{\sqrt{1-\alpha}}{1-\alpha}\left(k + w - c - \beta(H - E[D])\right) \end{pmatrix} \equiv \boldsymbol{B'}.$$

Notice that $k + w - c - \beta(H - E[D])$>0, because for the ILE to exist it must be that $k + w - \beta(H - E[D]) > 2B/3 + c/2$ (see Table 2) and 2*B*/3+*c*/3 > *c* because *B* >*c*. According to Table 2 and our assumption that *B* > *c*, we consider only cases in which $B \in \left(c,\ 1.5(k+w) - 1.5\beta(H - E[D]) - 0.5c\right) \equiv \mathbf{B}^*$. To complete the proof we show that this range, $\boldsymbol{B}^*$ is contained within ***B'*** (Details are available from authors by request)

Proof of Proposition 5: We can rewrite profits as $\pi_1 = L(w-c) - \alpha k(H-L) + (1-\alpha)(Q_1 - L)^2$ and $\pi_2 = Q_2^{\ 2}$. Both suppliers' profits are increasing in *B* because Q_1^0 and Q_2^0 are increasing in *B* and the terms that depend on β are not functions of *B*. In addition $\partial\pi_2/\partial B > \partial\pi_1/\partial B$ if and only if *B* >*k*+*w*-β(*H*-E[*D*]) because $\partial\pi_2/\partial B - \partial\pi_1/\partial B = 2\alpha\left(B - k - w + \beta(1-\alpha)(H - L)\right)/(3+\alpha)$.