Online Addendum for "Dynamic Procurement, Quantity Discounts, and Supply Chain Efficiency"

Feryal Erhun • Pınar Keskinocak • Sridhar Tayur

Department of Management Science and Engineering, Stanford University,

Stanford, CA 94305-4026, USA

School of Industrial and Systems Engineering, Georgia Institute of Technology,

Atlanta, GA 30332-0205, USA

Tepper School of Business, Carnegie Mellon University,

Pittsburgh, PA 15213, USA

ferhun@stanford.edu • pinar@isye.gatech.edu • stayur@andrew.cmu.edu

A.1. Limited Capacity Case

In this section, we consider the case where the supplier has a capacity of K units that he can sell throughout N-periods. Let Q_N correspond to the total production quantity of an N-period uncapacitated game. For the N-period limited capacity case, when the capacity is "tight," i.e., $C_S \leq \frac{a}{4b}$, or when the capacity is "abundant," i.e., $C_S \geq Q_N$, the results are straightforward and intuitive. In the first case, as the capacity is tight, the supplier does not change his price through the game, so the N-period game is equivalent to a single period game. When $C_S \geq Q_N$, the problem is equivalent to an unlimited capacity game (Proposition 1). What happens in between these extremes is more interesting. Our main result is as follows.

Proposition A.1 *The SPNE for the N-period capacitated model for $C_S < Q_N$ is as follows: Let $N^* \in \{1, 2, \cdots, N\}$ be such that $Q_{N^*-1} < C_S \leq Q_{N^*}$. For the first $N - N^*$ periods, the supplier and the buyer do not play the game. In the last N^*-periods, they play the following N^*-period game.*

For $i = 1, 2, \cdots, N^* - 2$:

$$q_{N-N^*+1} = C_S - \sum_{j=N-N^*+2}^{N} q_j, \qquad q_N = \tfrac{a}{2b} - C_S, \qquad q_{N-i} = \left(\frac{2i}{2i+1}\right) q_{N-i+1},$$

$$w_{N-N^*+1} = \prod_{i=1}^{N^*-1} \left(\frac{2i+1}{2i}\right) w_N, \quad w_N = 2\left(\tfrac{a}{2} - bC_S\right), \quad w_{N-i} = \left(\frac{2i+1}{2i}\right) w_{N-i+1}.$$

Proof: When $C_S < Q_N$, let N^* be the maximum N such that $Q_{N^*-1} < C_S \leq Q_{N^*}$. Then for the N^*-period game, by backward induction we can show the strategy given in the proposition is optimal. The proof directly follows from the proof of Proposition 1 after observing that $q_{N-N^*+1} = C_S - \sum_{j=N-N^*+2}^{N} q_j$, and $w_N = 2\left(\frac{a}{2} - bC_S\right)$.

As the total production quantity is equal to the available capacity, it is not possible to increase the total supply chain profit anymore. If we increase the number of periods, the best the supplier can do is to set capacity prices that are on average higher. However, as the total supply chain profit is constant, the buyer will be worse off in this case; hence she will not cooperate with the supplier forcing the play of an N^*-period game by not procuring in the other periods. □

When $C_S < Q_N$, depending on the capacity, the supplier and the buyer play an N^*-period game ($N^* \leq N$), the supplier sells all the capacity, and the supply chain is coordinated. Hence, contrary to the unlimited capacity case, increasing the number of trading periods beyond N^* does not increase the profits of either player. The total profits for the buyer and the supplier can be maximized in a finite number of trading periods, N^*, which depends on the capacity of the supplier.

A.2. Information Asymmetry

In practice, the buyer is closer to the end-consumer market and may have more information about the demand compared to the supplier. In this section, we consider information asymmetry between the supplier and the buyer regarding the market potential. According to the supplier, the market potential can either be "high," a_h, with a probability of α, or "low," a_l, with a probability of $1-\alpha$. The buyer, on the other hand, knows the exact value of the market potential. For simplicity, we assume that $b_i = 1$, $i = l, h$.

If the players interact only once ($N = 1$), the outcome of this asymmetric information game is as follows:

Proposition A.2 *The unique pure-strategy perfect Bayesian equilibrium of the single-period game with asymmetric market-potential information is as follows:*

$$w = \begin{cases} \frac{\bar{a}}{2} & \frac{a_l}{a_h} \geq \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}} \\ \frac{a_h}{2} & \frac{a_l}{a_h} \leq \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}} \end{cases}$$

where $\bar{a} = \alpha a_h + (1-\alpha)a_l$. *The buyer procures* $q_i = \left(\frac{a_i - w}{2}\right)^+$, $i = l, h$, *units of capacity.*

Proof: The buyer's best-response function is $q_i = \left(\frac{a_i - w}{2}\right)^+$. In the equilibrium, $w < a_h$. Hence, there are two cases to analyze: (1) $w < a_l$ and (2) $a_l \leq w < a_h$. If we solve for the supplier's optimization problem in the first case, we achieve $w = \bar{a}/2$. As $w < a_l$ in this case, the parameters should satisfy the following condition: $a_l/a_h \geq \alpha/(1+\alpha)$. Similarly, the analysis of the second case provides the solution $w = a_h/2$. For this case to be feasible, the parameters should satisfy the following condition: $a_l/a_h \leq 1/2$. The supplier's respective (expected) profits are $\bar{a}^2/8$ and $\alpha a_h^2/8$ under these two options. We see that the supplier prefers the former to the latter if:

$$\frac{\bar{a}^2}{8} \geq \frac{\alpha a_h^2}{8} \Rightarrow \bar{a} \geq \sqrt{\alpha} a_h \Rightarrow \frac{a_l}{a_h} \geq \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}},$$

which satisfies $\alpha/(1+\alpha) \leq \sqrt{\alpha}/(1+\sqrt{\alpha}) \leq 1/2$. □

If the supplier knew the market potential, he would set $w_i = \frac{a_i}{2}$, $i = l, h$. When there is information asymmetry, the relative values of a_l and a_h determine the equilibrium. When a_l and a_h are close to each other, the supplier sets the price based on the expected value $\bar{a}$. However, when a_h is considerably higher than a_l, the supplier sets the prices based on a_h and ignores the "low" state altogether. It is easy to see that the buyer has an incentive to reveal the market potential (her type) in the "low" state, but not in the "high" state.

Proposition A.3 summarizes the outcome of this asymmetric information game if the players interact twice ($N = 2$):

Proposition A.3 *A pure-strategy perfect Bayesian equilibrium of the two-period game with asymmetric market-potential information is as follows:*

$$w_1 = \begin{cases} > a_h & (I) \\ \frac{7\bar{a}}{16} + \frac{a_l}{8} & (II) \\ \frac{(a_h - a_l)\alpha(2-\alpha)}{2(1-\alpha)} & (III) \end{cases}$$

The supplier's posterior belief $\bar{\alpha}$ is equal to 1 if $q_1 > q_{1l}$ and to α otherwise, where $q_{1l} = \left(\frac{2a_l - \bar{a}}{6} - \frac{2w_1}{3}\right)^+$.

By choosing (I), the supplier effectively eliminates the first period. He can choose (II) and (III) if the conditions in equations (A.1) and (A.2) (see the proof of the proposition), respectively, are satisfied. The supplier compares (I), (II), and (III) and chooses the one with highest expected profits. Let $\bar{\bar{a}} = \bar{\alpha} a_h + (1-\bar{\alpha}) a_l$. We have, $q_{2i} = \left(\frac{a_i - w_2}{2} - q_1\right)^+$ and

$$w_2(q_1) = \begin{cases} \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^+ & \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^2 \geq \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2 \\ \left(\frac{a_h}{2} - q_1\right)^+ & \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^2 \leq \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2 \end{cases}$$

Proof: We solve for the perfect Bayesian equilibrium in this extensive-form game of incomplete information in pure strategies. We say that the buyer is of type i if the demand state is i, $i = l, h$.

Given her first-period procurement quantity (q_1), the buyer's second-period procurement quantity when she is of type i is $q_{2i} = \left(\frac{a_i - w_2}{2} - q_1\right)^+$, $i = l, h$. Given his posterior belief that the buyer is of "high" type ($\bar{\alpha}$) and q_{2i}, the supplier's second-period problem can be formulated as follows:

$$\max_{w_2 \geq 0} \quad \bar{\alpha} w_2 \left(\frac{a_h - w_2}{2} - q_1\right)^+ + (1 - \bar{\alpha}) w_2 \left(\frac{a_l - w_2}{2} - q_1\right)^+,$$

which leads to the following outcome:

$$w_2(q_1, \bar{\alpha}) = \begin{cases} \frac{\bar{\bar{a}}}{2} - q_1 & \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^2 > \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2 \\ \frac{a_h}{2} - q_1 & \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^2 < \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2 \\ \text{either } \frac{\bar{\bar{a}}}{2} - q_1 \text{ or } \frac{a_h}{2} - q_1 & \left(\frac{\bar{\bar{a}}}{2} - q_1\right)^2 = \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2 \end{cases}$$

where $\bar{\bar{a}} = \bar{\alpha} a_h + (1 - \bar{\alpha}) a_l$.

"Full Revelation": In order to solve for the first-period outcome, let us first consider the possibility of "full revelation," i.e., the buyer of type i is willing to reveal her type to the supplier. In this case $q_{1h} \neq q_{1l}$ and the supplier's posterior belief can be chosen as follows:

$$\bar{\alpha}(q_1) = \begin{cases} 1 & q_1 > q_{1l} \\ 0 & q_1 \leq q_{1l}. \end{cases}$$

Hence, the buyer's and the supplier's second period best-response functions are $q_{2i} = \left(\frac{a_i - w_2}{2} - q_{1i}\right)^+$ and $w_{2i} = \left(\frac{a_i}{2} - q_{1i}\right)^+$. Assuming $q_{1i} \leq \frac{a_i}{2}$, the type i buyer's first-period problem can be formulated as follows:

$$\max_{0 \leq q_{1i} \leq \frac{a_i}{2}} \quad \left(\frac{3a_i}{4} - \frac{q_{1i}}{2}\right)\left(\frac{a_i}{4} + \frac{q_{1i}}{2}\right) - w_1 q_{1i} - \frac{1}{2}\left(\frac{a_i}{2} - q_{1i}\right)^2.$$

Using the first-order condition (FOC), we get $q_{1i} = \frac{a_i}{2} - \frac{2w_1}{3}$. The FOC satisfies the bounds when $w_1 \leq \frac{3a_l}{4}$. However, in this case the second-period price is not type dependent, i.e., $w_{2i} = \frac{2w_1}{3}$. Therefore, there is a huge disincentive for the "high" type to reveal her type to the supplier, since the second-period price is lower. Hence, the supplier cannot set a price such that $w_1 \leq \frac{3a_l}{4}$ that leads to "full revelation."

If the "low" type buyer would rather not procure any quantity in the first period (i.e., $w_1 \geq \frac{3a_l}{4}$), we can formulate the supplier's problem as follows:

$$\max_{\frac{3a_l}{4} \leq w_1 \leq \frac{3a_h}{4}} \quad \alpha\left(w_1\left(\frac{a_h}{2} - \frac{2w_1}{3}\right) + \frac{2w_1^2}{9}\right) + (1 - \alpha)\left(\frac{a_l^2}{8}\right).$$

Using FOC, we get

$$w_1 = \frac{9a_h}{16}.$$

The upper bound is satisfied. The lower bound is satisfied when $\frac{a_l}{a_h} \leq \frac{3}{4}$. Next, we need to check the incentive compatibility constraint of the "high" type:

$$\underbrace{(a_h - (q_{1h} + q_{2h}))(q_{1h} + q_{2h}) - w_1 q_{1h} - w_{2h} q_{2h}}_{\frac{19a_h^2}{256}} \geq \underbrace{(a_h - \hat{q}_{2h})\hat{q}_{2h} - w_{2l}\hat{q}_{2h}}_{\frac{(2a_h - a_l)^2}{16}},$$

where q_{2h} is the quantity that the "high" type buyer procures when the price is set for her own type and $\hat{q}_{2h}$ is the quantity that she procures when the price is set for the "low" type.

When $\frac{a_l}{a_h} \geq \frac{8-\sqrt{19}}{4}$, the incentive compatibility constraint is satisfied. However, as $\frac{a_l}{a_h} \leq \frac{3}{4}$, $w_1 = \frac{9a_h}{16}$ cannot satisfy both the incentive compatibility constraint and the lower bound at the same time. Therefore, the "high" type buyer will not be willing to reveal her type truthfully unless the supplier decreases the first-period price considerably to make her indifferent between single period and two-period models:

$$-w_1\left(\frac{a_h}{2} - \frac{2w_1}{3}\right) - \frac{2w_1^2}{9} + \left(\frac{a_h}{2} - \frac{w_1}{3}\right)\left(\frac{a_h}{2} + \frac{w_1}{3}\right) - \frac{(2a_h - a_l)^2}{16} = 0,$$

from which we get:

$$w_1 = \frac{3a_h}{4} - \frac{\sqrt{3}}{4}\sqrt{(a_h - a_l)(3a_h - a_l)}.$$

However, in this case $w_1 \leq \frac{3a_l}{4}$, hence even the low type has an incentive to buy capacity in the first period. It is straightforward from the above argument that $w_1 = \frac{3a_l}{4}$ will not satisfy the incentive compatibility constraint. Hence, "full revelation" is not possible for the $N = 2$ game.

"No Revelation": It is also possible to target a "no revelation" equilibrium, i.e., for both types the buyer procures the same quantity in the first period. As the "low" type does not have an incentive to buy the "high" type's quantity, this means that in this case the "high" type will buy the "low" type's quantity. Therefore, the supplier's posterior belief can be chosen as follows:

$$\bar{\alpha}(q_1) = \begin{cases} 1 & q_1 > q_{1l} \\ \alpha & q_1 \leq q_{1l}. \end{cases}$$

First, let us assume $\left(\frac{\bar{a}}{2} - q_1\right)^2 < \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2$. Then, in the second period the supplier does not serve the "low" type buyer. It is easy to see that the supplier needs to quote a very low price in the first period in order to convince the "low" type buyer to procure. However, this solution will be dominated by $N = 1$.

Assuming $\left(\frac{\bar{a}}{2} - q_1\right)^2 > \bar{\alpha}\left(\frac{a_h}{2} - q_1\right)^2$, we can formulate the "low" type buyer's first-period problem as follows:

$$\max_{q_{1l} \geq 0} \quad \left(\frac{2a_l - \bar{a} + 2q_{1l}}{4}\right)\left(\frac{2a_l + \bar{a} - 2q_{1l}}{4}\right) - w_1 q_{1l} - \left(\frac{\bar{a}}{2} - q_{1l}\right)\left(\frac{2a_l - \bar{a} - 2q_{1l}}{4}\right).$$

Using FOC, we get $q_{1l}(w_1) = \frac{2a_l+\bar{a}}{6} - \frac{2w_1}{3}$. The FOC satisfies the non-negativity constraint when $w_1 \leq \frac{2a_l+\bar{a}}{4}$. Furthermore, the non-negativity condition of the second-period price and quantities are also satisfied. Note that we still have to verify that $\left(\frac{\bar{a}}{2} - q_{1l}\right)^2 > \bar{\alpha}\left(\frac{a_h}{2} - q_{1l}\right)^2$. Assuming this is the case, we can now formulate the supplier's problem:

$$\max_{0 \leq w_1 \leq \frac{2a_l+\bar{a}}{4}} \quad w_1\left(\frac{2a_l+\bar{a}}{6} - \frac{2w_1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{\bar{a}-a_l+2w_1}{3}\right)^2.$$

Using FOC, we get

$$w_1^* = \frac{7\bar{a}}{16} + \frac{a_l}{8}.$$

First let us verify that the "high" type buyer's incentive-compatibility constraint is not violated at w_1^*:

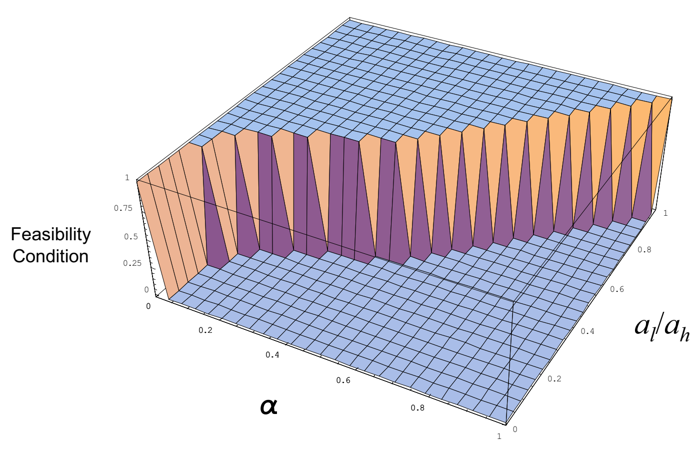
$$-w_1^* q_{1l}(w_1^*) - \left(\frac{\bar{a}}{2} - q_{1l}(w_1^*)\right)\left(\frac{2a_h-\bar{a}}{4} - \frac{q_{1l}(w_1^*)}{2}\right) + \left(\frac{2a_h+\bar{a}}{4} - \frac{q_{1l}(w_1^*)}{2}\right)\left(\frac{2a_h-\bar{a}}{4} + \frac{q_{1l}(w_1^*)}{2}\right) \geq$$
$$-w_1 q_{1h}(w_1^*) - \left(\frac{a_h}{2} - q_{1h}(w_1^*)\right)\left(\frac{a_h}{4} - \frac{q_{1h}(w_1^*)}{2}\right) + \left(\frac{3a_h}{4} - \frac{q_{1h}(w_1^*)}{2}\right)\left(\frac{a_h}{4} + \frac{q_{1h}(w_1^*)}{2}\right),$$

where $q_{1h}(w_1) = \frac{a_h}{2} - \frac{2w_1}{3}$. When $9(\frac{a_l}{a_h} - \alpha) + (1 - \frac{a_l}{a_h})\alpha^2 \geq 0$, the "high" type buyer does not have an incentive to reveal her type, hence w_1^* is incentive compatible. When w_1^* is incentive compatible, the upper bound is automatically satisfied, i.e., $\bar{a} \leq 2a_l$. Therefore, when

$$\left(\frac{\bar{a}}{2} - q_{1l}(w_1^*)\right)^2 > \alpha\left(\frac{a_h}{2} - q_{1l}(w_1^*)\right)^2 \quad \text{and} \quad 9\left(\frac{a_l}{a_h} - \alpha\right) + \left(1 - \frac{a_l}{a_h}\right)\alpha^2 \geq 0, \tag{A.1}$$

w_1^* is a candidate for the equilibrium. Figure A.1 shows that such parameters do exist (the feasible region has a value of 1).

Figure A.1: Feasibility conditions for the "no revelation" equilibrium.



When $9(\frac{a_l}{a_h}-\alpha)+(1-\frac{a_l}{a_h})\alpha^2<0$, the supplier needs to increase the first-period price in order to satisfy the incentive compatibility constraint of the buyer and the resulting w_1 is:

$$w_1^{IN}=\frac{(a_h-a_l)\alpha(2-\alpha)}{2(1-\alpha)}.$$

When $\frac{a_l}{a_h}\geq\frac{\alpha(3-\alpha)}{3-\alpha^2}$, w_1^{IN} satisfies the upper and lower bounds, as well as the nonnegativity constraints of second-period price and quantities. Therefore, when

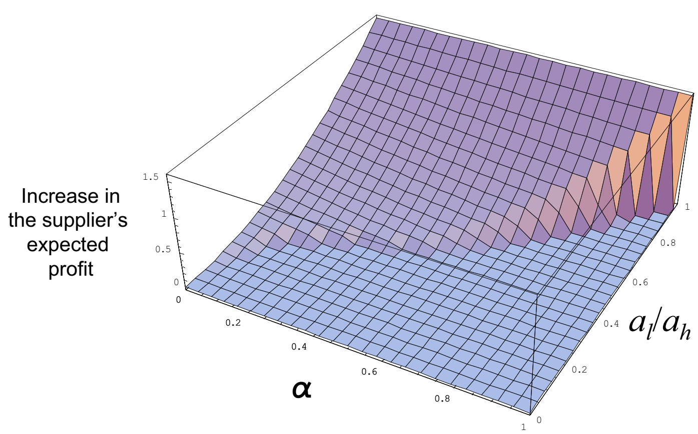
$$\left(\frac{\bar{a}}{2}-q_{1l}(w_1^{IN})\right)^2>\alpha\left(\frac{a_h}{2}-q_{1l}(w_1^{IN})\right)^2,\ 9\left(\frac{a_l}{a_h}-\alpha\right)+\left(1-\frac{a_l}{a_h}\right)\alpha^2<0,\ \frac{a_l}{a_h}\geq\frac{\alpha(3-\alpha)}{3-\alpha^2},\quad\text{(A.2)}$$

w_1^{IN} is a candidate for the equilibrium.

To summarize, the supplier can either quote w_1^* (if conditions in (A.1) are satisfied) or w_1^{IN} (if conditions in (A.2) are satisfied) to induce a "no revelation" equilibrium. Another alternative is to quote a very high price in the first period, which leads to a single-period solution. The supplier compares his profits under these three cases and chooses the one that maximizes his profit. □

With the goal of understanding the impact of multiple trading periods in an asymmetric information setting, consider Figure A.2. Based on the results of Proposition A.3, for different values of a_l and α, Figure A.2 plots the increase in the supplier's expected profits due to the additional period.

Figure A.2: The increase in the supplier's expected profits due to the additional period ($a_h=10$).



When a_l is relatively low compared to a_h, the supplier does not benefit from the additional period. For higher values of a_l, there are values of α for which dynamic procurement improves the system performance. Hence, under asymmetric information the additional trading period may continue to enable dynamic procurement depending on the relative values of a_l and a_h. However, our main result may also be reversed under asymmetric information.