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Online Supplement for

"Optimal Contract Design for Mixed Channels Under Information Asymmetry"

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Appendix

Proof of Proposition 1(a)

$$\pi^I = p_1 d_1 + (p_2 - c_v)d_2 = p_1(a - p_1 + r(p_2 - v)) + (p_2 - \frac{\eta v^2}{2})(a - p_2 + v + p_1 r)$$

Then we take first order condition with respect to p_1, p_2 and v, and set them equal to zero, respectively. After that, solving these three equations simultaneously, we can get the desired result.

Proof of Proposition 1(b)

$$\pi_R^F = (p_2^* - w - C_v^*)d_2 + L^F = \pi_{\bar{R}}$$

$$\Rightarrow L^F = \pi_{\bar{R}} - (\frac{2a\eta + 1}{4\eta})^2$$

$$\pi_M = \begin{cases} \frac{a}{4\eta} + \frac{1}{16\eta^2} + \frac{a^2}{2(1-r)} - \pi_{\bar{R}} & \underline{\eta} \le \eta \le N \quad (a) \\ \pi_{\bar{M}} & N \le \eta \le \bar{\eta} \quad (b) \end{cases}$$

Setting (a) = (b), we get

$$N = \frac{1}{-2a + 2\sqrt{4\pi_{\bar{M}} + 4\pi_{\bar{R}} - a^2 \frac{1+r}{1-r}}} \quad where \; \pi_{\bar{M}} + \pi_{\bar{R}} \ge \frac{a^2(1+r)}{4(1-r)}$$

Due to N>0, we only keep the one with positive value.

<u>Proof of Proposition 2(a)</u>

The equation (10), (11) and (12) can be written as:

$$\max \int_{\underline{\eta}}^{N} m(\eta) d\eta + \Phi(N)$$

$$s.t.$$

$$\dot{L}(\eta) = g_1(\eta), \quad \dot{w}(\eta) = g_2(\eta), \quad \dot{p}_1(\eta) = g_3(\eta)$$

This is obtained by making the following variable substitution:

$$m := (p_1 d_1(p_2^r) + w d_2(p_2^r) - L) f$$

$$= \{p_1[(1+\frac{r}{2})a + (\frac{r^2}{2} - 1)p_1 - \frac{r}{4\eta}] + w(\frac{a}{2} + \frac{1}{4\eta} - \frac{w}{2}) + rwp_1 - L\} f,$$

$$g_1 = (\frac{1}{4\eta} + \frac{a + rp_1 - w}{2})(u_1 - ru_2), \; g_2 = u_1, \; g_3 = u_2,$$

$$\Phi(N) = \pi_{\bar{M}}(1-F).$$

Using the multiplier equations gives following results:

$$\dot{\lambda}_1 = f \text{ and } \lambda_1 = F \tag{2.1}$$

$$\dot{\lambda}_2 = -(-w + \frac{a}{2} + \frac{1}{4\eta} + rp_1) f + \frac{\lambda_1}{2}(u_1 - ru_2) \tag{2.2}$$

$$\dot{\lambda}_3 = -[a(1+\frac{r}{2}) + 2p_1(\frac{r^2}{2} - 1) - \frac{r}{4\eta} + rw] f - \frac{\lambda_1 r(u_1 - ru_2)}{2} \tag{2.3}$$

Using the optimality conditions gives following results:

$$\lambda_1(\frac{1}{4\eta} + \frac{a + rp_1 - w}{2}) + \lambda_2 = 0 \tag{2.4}$$

$$-r\lambda_1(\frac{1}{4\eta} + \frac{a + rp_1 - w}{2}) + \lambda_3 = 0 \tag{2.5}$$

Taking derivative on both sides of (2.4) and using (2.1), we get

$$\dot{\lambda}_2 = -(\frac{1}{4\eta} + \frac{a + rp_1 - w}{2})f - F(-\frac{1}{4\eta^2} + \frac{r}{2}u_2 - \frac{u_1}{2}) \quad (2.6)$$

Solving (2.6) with (2.2), we get $\frac{F}{2\eta^2} = fw - frp_1$ (2.7)

Taking derivative on both sides of (2.5) and using (2.1), we get

$$\dot{\lambda}_3 = rf(\frac{1}{4\eta} + \frac{a + rp_1 - w}{2}) + rF(-\frac{1}{4\eta^2} + \frac{r}{2}u_2 - \frac{u_1}{2}) \quad (2.8)$$

Solving (2.8) with (2.3), we get $\frac{rF}{4\eta^2} = f[a + ar + (\frac{3r^2}{2} - 2)p_1 + \frac{rw}{2}]$ (2.9)

Solving (2.7) and (2.9) together, we get desired result

$p_1^A = \frac{a}{2(1-r)}, w^A = \frac{F}{2f\eta^2} - \frac{ra}{2(1-r)}$ and

$$\dot{L}(\eta) = g_1(\eta) = (\frac{1}{4\eta} + \frac{a + rp_1 - w}{2})(u_1 - ru_2) = (\frac{1}{4\eta} + \frac{a + rp_1 - w}{2})\dot{w}.$$

Using the transversality conditions if N is free

$m(N) + \lambda_1(N)g_1(N) + \lambda_2(N)g_2(N) + \lambda_3(N)g_3(N) + \Phi_N = 0$ at N

we get the following results: $(p_1 d_1 + wd_2 - L - \pi_{\bar{M}})f = 0$. Because $f \neq 0$,

$p_1 d_1 + wd_2 - L - \pi_{\bar{M}}$ must equals to 0. The manufacturer can make

$p_1 d_1 + wd_2 - L - \pi_{\bar{M}} \geq 0$ binding at η_1, $\eta_1 = N$. Then substitute p_1 and w with p_1^A(N) and w^A(N), we get that $L(N)^A$ satisfies

$$-\frac{F^2}{8N^4 f^2} + \frac{aF}{4N^2 f} + \frac{F}{8N^3 f} + \frac{a^2(1+r)}{4(1-r)} - L(N)^A = \pi_{\bar{M}}.$$

η_0 can be solved by let $(p_2^A - w^A - c_v{}^A)d_2 + L^A \geq \pi_{\bar{R}}$ binding at η_0.

Proof of Proposition 3

(i) Manufacturer: Adding $\pi_R^I + \pi_M^I \geq \pi_R^A + \pi_M^A$ from the proposition 3(ii) and $\pi_R^A \geq \pi_R^I$ from the proposition 3(iii), we can get $\pi_M^I \geq \pi_M^A$.

(ii) Retailer: Under (I), the retailer earns her reservation profit through the whole range of η. Under (A), it is always higher or equal to her reservation profit. Therefore,

$\pi_R^I \le \pi_R^A$ for all η. As an example, suppose follows a Uniform distribution where

$F = \dfrac{\eta - \eta_0}{\eta_3 - \eta_0}$, $f = \dfrac{1}{\eta_3 - \eta_0}$. Then, $\pi_R^A = \dfrac{a}{2} + \dfrac{\eta_0}{4\eta^2}$ which is decreasing with η to a value of

$\pi_{\bar{R}}$ at $\eta = N^A$. At the same time, profit for the case I is constant at $\pi_{\bar{R}}$ for all η, so we have $\pi_R{}^I \le \pi_R{}^A$.

(iii) The supply chain: The supply chain profit under (I) $\pi^I = \dfrac{a}{4\eta} + \dfrac{1}{16\eta^2} + \dfrac{a^2}{2(1-r)}$ and the supply chain profit under (A) is $\pi^A = \pi_R^A + \pi_M^A = \dfrac{a}{4\eta} + \dfrac{1}{16\eta^2} + \dfrac{a^2}{2(1-r)} - \dfrac{F^2}{16\eta^4 f^2}$

Obviously, $\pi^I > \pi^A$.

Proof of Proposition 6

Under uniform distribution:

$d_2^I = \dfrac{a}{2} + \dfrac{1}{4\eta}$ and $d_2^A = \dfrac{a}{2} + \dfrac{1}{4\eta} - \dfrac{F}{4f\eta^2}$. So $d_2^I \ge d_2^A$.

$d_1^I = \dfrac{a}{2} - \dfrac{r}{4\eta}$ and $d_1^A = \dfrac{a}{2} - \dfrac{r}{4\eta} + \dfrac{rF}{4f\eta^2}$. So $d_1^I \le d_1^A$.