Manufacturing Complexity: A Quantitative Measure

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ABSTRACT

Manufacturing complexity is not easily defined. The ability to measure manufacturing complexity will permit benchmarking of systems, and allow assessment of system design changes on complexity. This paper will classify existing complexity measures, identify the essential elements of a good measure, and propose a measure of manufacturing complexity.

Introduction

A manufacturing operation is a system made up of many subsystems, or parts. Due to the interrelationships of the many parts, academics and practitioners, alike, will often say that a manufacturing system is complex. But, usually they do not give a precise definition of complexity.

Complexity is difficult to define precisely. A general definition of complexity is that a complex system is one, which has a large number of parts, whose relationships are not simple {Simon, 1962 #1}. Note that the parts themselves, may be simple, but that their relationships are not simple {Pippenger, 1978 #2}.

Complexity is different from subtlety and it does not exist where there is either complete order or complete disorder. Subtlety exists when consequences of interventions are not obvious to most system participants {Senge, 1990 #3}. Subtle systems have few parts in them, but there are many layers of relations between the parts. On the other hand, complexity has many parts with numerous relations between the parts, and each of these relationships is obvious {Cooper, 1992 #4}.

There is no complexity in systems, which are completely organized. While all of the relationships in a completely organized system are obvious, there are a limited number of relationships between the parts. There is also no complexity at the other end of the continuum where everything is completely disordered. All the elements in completely disordered systems act in unpredictable ways. While there may be many elements in a disordered system, but their unpredictability means there is no obvious relationship between the elements, so the system is not complex {Cooper, 1992 #4}. Complex systems exist only when there are ordered relationships between the elements in the system. So, complexity exists in those systems, which are on the continuum between complete disorder and total order {Gell-Mann, 1995 #5}.

Why Use Complexity to Examine Manufacturing Management?

Manufacturing's systems are complex. Manufacturing systems have many elements. The elements have obvious, but non-simple relationships to each other. The elements in the typical manufacturing system are not completely organized, and they are not completely disordered.

Frizelle and Woodcock (1995) argue that measuring manufacturing complexity provides a useful metric for improvement. They argue that systems with higher complexity have more problems than systems with lower complexity. So, by measuring the system's complexity, the managers can identify problems in the system that are hindering the production flow.

In addition, if a manufacturing complexity measure could be easily computed, it would allow researchers to compare different system designs / structures to one another. And, it would allow system managers to determine how system changes (e.g., policy and/or process changes). Finally, the measure will allow a precise measurement of how changes in complexity in one system influence other systems in the organization.

Requirements for a Useful Complexity Measure

While researchers may be willing to use measures that are difficult to compute and require data that is difficult to obtain, most practicing managers want to be able to obtain the data quickly and be able to use the data in a clear, step-by-step analysis.

Typically, managers are not willing to apply a measure if they must invest hundreds of man-hours for data collection each time they want to compute it. This means that a complexity measure must use objective data, that can be obtained reliably by different observers of the system.

For any system measure to be useful, it must of course be a valid measure of the system being studied. Frizelle (1996) argued that in addition to validity, a useful complexity measure needs to be composed of separable, additive components. By being separable and additive the manufacturing complexity measure would then allow easy analysis of managers' interventions on the system.

Separating the complexity measure into two components simplifies the computation of the measure. One is used to measure the system structure and the other to measure the system uncertainty. Frizelle and Woodcock (1995) used static complexity as a measure of complexity due to the system design, while dynamic complexity was seen as the result of the uncertainties in the system while it is operating (e.g., machine breakdowns).

Deshmukh, Talavage and Barash (1998) provided a clear definition of static complexity and dynamic complexity. Static or structural complexity is a "function of the structure of the system, connective patterns, variety of components, and the strengths of interactions" (p. 645). So, static complexity measures how the factory is structured (e.g., number of products, number of processes/machines). On the other hand, dynamic complexity is a measure of "the unpredictability in the behavior of the system over a time period" (p. 645). A common example of dynamic complexity is a machine breakdown.

This means that dynamic complexity is an obstacle to achieving the system's goals {Frizelle, 1996 #14}.

There is obviously not a clear separation between static and dynamic complexity, since dynamic complexity is determined at least in part by the design of the system or the static complexity that exists. For example, a manufacturing system that includes a comprehensive maintenance system, should have fewer breakdowns and consequently less dynamic complexity.

The goal of this paper is to develop succinct measures of manufacturing complexity. These complexity measures should provide the practitioner a tool to compare system designs and to measure improvement. It should also allow researchers to quantitatively analyze the relationships between system design and system performance.

Literature Review

Complexity

Two major approaches to measuring complexity have been reported in the literature. One approach counts parts of the system, such as the number of components {Pippenger, 1978 #2} {Klir, 1985 #17}. The second approach views complexity as a cognitive phenomenon. This cognitive approach considers systems that take longer to describe as being more complex than systems, which can be described in fewer words. The argument is that the longer the required description for a system, the more interrelationships exists, and therefore the greater complexity {Löfgren, 1977 #19}. These approaches do not actually contradict each other, they differ only in what they chose to count.

Klir (1985) reconciled these approaches by stating that “the complexity of a system should be proportional to the amount of information required to describe the system.” And he went on to measure complexity as the number of entities in the system and the variety of the interrelationships of these entities. By variety, Klir meant that the unique interdependencies of the system needed to be separated from those interdependencies that were “common”. As commonality increases, variety decreases, and complexity decreases.

Colinescu et al. (1998) divided complexity into five categories. The first four categories were sub-categories of static complexity and the fifth was a measure of dynamic complexity. The categories are:

- Product structure
- Structure of the shop or plant
- Planning and scheduling functions
- Information flow
- Dynamism, variability and uncertainty of the environment.

Cooper, et al. (1992) considered only elements of static complexity in their measure. They counted the changes in product mix, process mix and process-flow characteristics in their measure. $TNDE(j)=\sum V(i,j)PPI(i)\ PFLOW(i,j)$, where PP(i) is the product process index of the i^{th} chip type; V(i,j) is the volume of chip type i in the j^{th} period; and PFLOW(i,j) is the flow index. They argued that a good complexity measure should consider only first order effects to clearly separate complexity from subtly. For example, while the variety of products could be considered, they did not separately measure how the product variety influences the inventory level.

Much of the research developing complexity measures has developed measures focused on one aspect of manufacturing complexity. For example, measures of part commonality and bill-of-materials complexity, and product complexity have been developed.

Collier (1981) investigated the effects of part commonality, an element of product structure complexity, on system performance, and subsequently (1982) its effect on the safety stock level. Collier's results indicate that part commonality does affect system performance. Wacker and Treleven (1986) evaluated and developed various part commonality indices for use in future research. Hillier (1999) sought to extend research on part commonality to a multi-period model for make-to-stock environments.

Benton and Srivastava (1985) examined bill-of-material complexity in relation to lot-sizing policies. Their measure of complexity included product structure breadth (average components per parent) and depth (number of levels). They found that complexity had little impact on lot sizing. Adding a measure of routing complexity (number of operations), they investigated the interaction of routing complexity with lot sizing and inventory storage capacity to determine systems cost in multi-level systems (1993). They found that when product structures became more complex, total system costs increased.

Sum et al. (1993), produced research relating product structure complexity and various lot-sizing rules in a multi-level system. Three components defined their complexity measure – number of items (all levels), number of levels, and a part commonality index. Fry et al. (1989), tested product structure complexity (breadth and depth) and sequencing (scheduling) rules related to performance in an assembly shop.

Krajewski et al. (1987) included the degree of part commonality, the number of levels in the bill of materials, and the number of components per parent (depth) in an exploratory study of factors influencing manufacturing performance. Product structure was identified as having a significant influence on the performance in a kanban system.

Some researchers have been developing complexity measure for manufacturing derived from entropy measures used in information theory. In information theory, entropy is:

> "… a measure of the amount of information that is output by a source, or throughput by a channel, or received by an observer (per symbol or per second)." (Dictionary, 1996)

Mathematically, entropy is measured as ($H = \sum_{x \in a} px \log(1/p(x))$), where x is a symbol in the range of alphabet a, and p(x) is the probability of symbol x as the next letter.

Frizelle and Woodcock (1995) developed an entropy based measure of manufacturing complexity. Deshmukh et al. (1998) also proposed an entropic measure to evaluate the static complexity in a machine shop (no assembly). Included in their measure were number of manufactured parts, number of machines, specific routings for each part, flexibility of machines and routings, and volume mix. They related their measure to the average waiting time. They concluded that higher static complexity results in lower waiting times, which they suggested is a result that is the opposite of what is expected from the lean manufacturing literature.

The entropy measure that both Frizelle and Woodcock (1996) and Desmukh et al. (1998) base their complexity measures on uses a base 2 log. They argued that the advantage of using a log was that it reduced the impact of adding one more item, part or relationship to the system. For example, the addition of one new end product to a firm that currently produces five end products increases that system’s complexity more than if the same addition were made to a firm producing 100 end products.

Measuring Complexity

Past literature reveals that existing measures of complexity are not inclusive and some have data requirements that are intensive. Many have focused on the elements of static complexity, while few have included the concepts of dynamic complexity. This paper proposes a quantitative measure for manufacturing complexity based upon the structural, or static, elements of a production system.

Dynamic complexity is not considered in this research, since dynamic complexity measures uncertainty during operations and this research focuses on system design.

The proposed measure is composed of two components – product structure complexity and routing complexity. The product structure of a system has been shown to have an impact on system performance {Krajewski, 1987 #18}; {Benton, 1993 #7}. Also, one component of product structure is the level of part commonality. Collier (1981) determined that part commonality affects system performance.

Part routings create static complexity. If there is only one routing and every part must take it, there is no complexity. However, if there are multiple routings for every part, then there is a high complexity level.

Product Structure Complexity Measure

There are four components of product structure complexity that seem to contribute to complexity. These are:

1. the number of end items,
2. the number of manufactured items in the end items' bill of materials,
3. the number of levels in a product structure, and
4. the degree of part commonality.

The number of end items and the number of manufactured items in the end items' bill of materials, shuld incrase the complexity of scheduling andmaterial control. The number of level in the product structure also independently contribute to the levelof complexity. This has been explored before in terms of the depth of the bill of materials {Krajewski, 1987 #18} {Benton, 1993 #7}. Intuitively, there are fewer relationships to manage when there are only two levels in the bill-of-materials, so that all of the relationships are from the component to the parent.

The degree of part commonality also influences static complexity. Higher levels of part commonality, should reduce complexity, since there will be fewer independent items which have relationships to other items.

In the measure proposed in this paper, the complexity associated with purchasing raw materials and components is not considered. The number of end-items and manufactured items only are included in the computation. Also, since the lowest level of a product structure consists solely of purchased items, we reduce the number of levels by one to eliminate the impact of these items.

Merely counting the number of end items and manufactured items gives equal weight to each. But, an end item produced infrequently, in low quantity should create less complexity than one produced frequently, in large quantities. So, the measure proposed here weights the end items, component items and the number of levels by the volume produced. The weighted average number of levels is measured as,

$$\frac{\sum_{i=1}^{e}\left[V_i \times \frac{\left(Levels(E_i)-1\right)+\sum_{j}^{|C_i|}\left(Levels(C_{ij})-1\right)}{|C_i|+1}\right]}{\sum_{i=1}^{e}V_i} \qquad equation\ (2),$$

where e is the number of distinct end-items, E_i represents the i^{th} end-item, $|C_i|$ is the number of manufactured components in the i^{th} end-item, C_{ij} represents the j^{th} manufactured component of the i^{th} end-item, and V_i represents the volume of requirements for the i^{th} end-item.

The influence of part commonality on complexity is measured using the total constant commonality index (TCCI) {Wacker, 1986 #22}. This index is a modification of the degree of commonality index (DCI) {Collier, 1981 #11}. However, the DCI does not have a limited range, while the TCCI measure provides a range from 0 (no commonality) to 1 (one item used everywhere). Wacker and Treleven's (1986) commonality index is:

$$TCCI = 1 - \frac{d-1}{\sum_{j=1}^{d}\phi_j - 1},$$

where d is the number of distinct component parts, and Φ_j represents the number of immediate parents for the j^{th} component.

The TCCI index was subtracted from 2 so that when there was complete commonality, it would become 1 and not contribute to complexity in our complexity measure. When there is no commonality, the term becomes 2, which serves to double the amount of complexity in our measure.

The resulting product structure complexity component is:

$$(E+C)\times\left\{\frac{\sum_{i=1}^{e}\left[V_i\times\frac{\left(Levels(E_i)-1\right)+\sum_{j}^{|C_i|}\left(Levels(C_{ij})-1\right)}{|C_i|+1}\right]}{\sum_{i=1}^{e}V_i}\right\}\times\left(2-TCCI\right) \qquad equation\ (3),$$

where E is the total number of end items and *C* is the total number of manufactured items.

Routing Complexity Measure

Routing complexity is composed of separate components. These are:

1. the number of end items,
2. the number of manufactured items in the end items' bill of materials, and
3. the number of steps in a routing.

The rationale for considering the complexity of the end items and the number of manufactured items in the bill-of-materials was given earlier. The number of steps in a routing also contributes independently to complexity. First, the number of steps in a routing is typically greater than the number of levels in a bill ofmaterials. Second, as the numbe rof steps increases, there ar emor relationships between the items and the resources used. Again, the volume produced with each routing influences the total complexity, so the complexity of the routing is volume weighted.

The volume weighted formula for the routing complexity is calculated by:

$$\frac{\sum_{i=1}^{e}\left[V_i \times \frac{Steps(E_i)+\sum_{j}^{|C_i|} Steps(C_{ij})}{|C_i|+1}\right]}{\sum_{i=1}^{e} V_i} \qquad equation\ (4),$$

where e is the number of distinct end-items, E_i represents the i^{th} end-item, $|C_i|$ is the number of manufactured components in the i^{th} end-item, C_{ij} represents the j^{th} manufactured component of the i^{th} end-item, and Q_i represents the requirements for the i^{th} end-item.

The resulting routing complexity measure is simply:

$$(E+C)\times\left\{\frac{\sum_{i=1}^{e}\left[V_i \times \frac{Steps(E_i)+\sum_{j}^{|C_i|} Steps(C_{ij})}{|C_i|+1}\right]}{\sum_{i=1}^{e} V_i}\right\} \qquad equation\ (5),$$

where E is the total number of end items and C is the total number of manufactured items.

Future Research

While the quantitative measure proposed in the paper has met the outline objectives, it is just a good starting point. The research can be extended by work on finding a measure for routing commonality. Additionally, other components of a system’s structure, i.e. layout, add to the static complexity. Incorporating valid quantifiable measures of numerosity and commonality would strengthen the measure.

Recognizing that complexity may impact performance, future research could use the proposed measure for complexity and test the relationship. Comparing a variety of manufacturing structures with similar and dissimilar complexity values, could be used to answer questions like,

1. How does simplification affect performance?
2. What is the relative impact of longer routings?
3. Is performance improved through outsourcing (shorter product structures)?

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